1. Consider the system \( \frac{dY}{dt} = AY \), where \( A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \).

(a) Find the eigenvalues of \( A \).

(b) Find the eigenvectors of \( A \).

(c) Find the general solution of the system.
(d) Find the solution of the system with the initial condition \( Y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \).

(e) Sketch the phase portrait, including the solution curve with the initial condition \( Y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \).
2. Consider the one-parameter family of linear systems given by

\[ \frac{dY}{dt} = \begin{pmatrix} a & a + 1 \\ a - 1 & a \end{pmatrix} Y. \]

(a) Sketch the corresponding curve in the trace-determinant plane.

(b) Identify which types of behaviors the system exhibits for which values of \( a \).
3. Suppose a block with mass 1 is attached to the end of a spring with spring constant 5. The block is subject to a damping force proportional to its velocity, with a damping coefficient 4. Finally, an external time-dependent force of \( \cos 2t \) acts on the block.

(a) Write a differential equation which models the behavior of the block.

(b) Find the general solution of your differential equation.
(c) Describe the long-term behavior of the block.

(d) Suppose that at time 0 the block is at rest and the spring is stretched so that the block is a distance 1 from its equilibrium position. Determine the position of the block for all times $t$. 