Homework 19

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by reflection across the plane $x + y + z = 0$. Find the matrix of T with respect to the basis $\mathcal{B} := \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$.

2. Let $S : \mathcal{P}_m(\mathbb{R}) \to \mathcal{P}_{m-1}(\mathbb{R})$ be defined by $Sf(x) = f'(x)$. Choose bases of $\mathcal{P}_m(\mathbb{R})$ and $\mathcal{P}_{m-1}(\mathbb{R})$, prove that they are in fact bases, and then find the matrix of S with respect to your choice of bases.

3. Show that
$$\langle p_1, p_2 \rangle := \int_0^1 p_1(t)p_2(t)dt$$
defines an inner product on $\mathcal{P}(\mathbb{R})$.