1. Show that similar matrices have the same eigenvalues.

2. Let $A \in M_n(\mathbb{C})$ have singular values $s_1, \ldots, s_n$, and define $B \in M_{2n}(\mathbb{C})$ by

$$B = \begin{bmatrix} 0 & A \\ A^* & 0 \end{bmatrix}.$$ 

Show that the eigenvalues of $B$ are $s_1, \ldots, s_n, -s_1, \ldots, -s_n$. *Hint:* Use the singular value decomposition of $A$.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by reflecting across the line making angle $\theta$ with the $x$-axis. Find the matrix of $T$ in a convenient basis, and then use it to find the matrix of $T$ with respect to the standard basis.

4. Let $S$ be an invertible linear map on a finite-dimensional vector space $V$ and $\{e_i\}_{i=1}^n$ a basis of $V$. Show that $\{Se_i\}_{i=1}^n$ is also a basis of $V$ and that

$$M(I, \{Se_i\}, \{e_i\}) = M(S, \{e_i\}).$$