1. Let $P$ be the plane

$$\begin{align*}
\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid 3x - 2y + z &= 0 \right\}.
\end{align*}$$

(a) Find a basis for $P$.

(b) Determine whether each of the following vectors is in $P$. For each one that is, give its coordinate representation in terms of your basis.

i. $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

ii. $\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$

iii. $\begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$

2. (a) Show that $B = (1, x, \frac{3}{2}x^2 - \frac{1}{2})$ is a basis of $P_2(\mathbb{R})$.

(b) Find the coordinate representation of $x^2$ with respect to $B$.

(c) Let $D : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the derivative operator. Find the coordinate representation of $D$ with respect to $B$ (i.e., with the same basis $B$ on both the domain and the codomain). Use it to calculate $\frac{d}{dx}[x^2]$. 