Math 307 Homework
November 18, 2015

1. Prove that if $\mathbb{F}$ is a field with only finitely many elements, then $\mathbb{F}$ is not algebraically closed.

   Hint: Find a polynomial over $\mathbb{F}$ such that every element of $\mathbb{F}$ is a root, and add 1.

2. Suppose that $\mathbf{x} \in \mathbb{F}^n$ is an eigenvector of $\mathbf{A} \in \mathrm{M}_n(\mathbb{F})$ with eigenvalue λ, and let $p(x)$ be any polynomial with coefficients in $\mathbb{F}$.

   (a) Prove that $\mathbf{x}$ is also an eigenvector of $p(\mathbf{A})$ with eigenvalue $p(\lambda)$.

   (b) Prove that if $p(\mathbf{A}) = \mathbf{0}$, then $p(\lambda) = 0$.