1. Prove that if $\mathbb{F}$ is a field with only finitely many elements, then $\mathbb{F}$ is not algebraically closed.

   Hint: Find a polynomial over $\mathbb{F}$ such that every element of $\mathbb{F}$ is a root, and add 1.

2. Suppose that $x \in \mathbb{F}^n$ is an eigenvector of $A \in M_n(\mathbb{F})$ with eigenvalue $\lambda$, and let $p(x)$ be any polynomial with coefficients in $\mathbb{F}$.

   (a) Prove that $x$ is also an eigenvector of $p(A)$ with eigenvalue $p(\lambda)$.
   (b) Prove that if $p(A) = 0$, then $p(\lambda) = 0$. 