1. Let $A \in M_n(\mathbb{C})$ and let $\varepsilon > 0$. Show that there is a $B \in M_n(\mathbb{C})$ with $n$ distinct eigenvalues such that $\|A - B\| \leq \varepsilon$.

*Hint:* First consider the case where $A$ is upper triangular, then use the Schur decomposition.

2. (a) Prove that if $A \in M_n(\mathbb{C})$ is upper triangular and normal, then $A$ is diagonal.

(b) Use this fact and the Schur decomposition to prove the spectral theorem for normal matrices.

3. Show that if $D : M_n(\mathbb{F}) \rightarrow \mathbb{F}$ is an alternating, multilinear function, then you can add any linear combination of the columns to any one column of a matrix $A$ without changing the value of $D(A)$. 