Math 307 Homework
November 4, 2015

1. Show that $\mathbf{A} \in \mathrm{M}_n(\mathbb{C})$ is unitary if and only if $\sigma_j = 1$ for $j = 1, \ldots, n$.

2. Show that if $\mathbf{A} = \mathbf{diag}(\lambda_1, \ldots, \lambda_n)$, then the singular values of $\mathbf{A}$ are $|\lambda_1|, \ldots, |\lambda_n|$ (though not necessarily in the same order).

3. Prove that for any $\mathbf{A} \in \mathrm{M}_{m,n}(\mathbb{C})$,

$$\|\mathbf{A}\| \leq \|\mathbf{A}\|_F \leq \sqrt{\min\{m, n\}} \, \|\mathbf{A}\| .$$

4. For each $z \in \mathbb{C}$, let $\mathbf{A}_z = \begin{bmatrix} 1 & z \\ 0 & 2 \end{bmatrix}$.

   (a) Find all the eigenvalues of $\mathbf{A}_z$, and show that they don't depend on z.

   (b) Show that the singular values of $\mathbf{A}_z$ do depend on z.
   Hint: You don't actually need to calculate the singular values of $\mathbf{A}_z$. Use something related to the singular values which is simpler to compute.