1. Show that
\[ \mathbb{Q}[\sqrt{2}] := \{ a + b\sqrt{2} : a, b \in \mathbb{Q} \} \]
is a field.

2. (addition to problem 4) Explain why it follows immediately from problem 4 that if \( a, b \in F \) and \( a \cdot b = 0 \), then either \( a = 0 \) or \( b = 0 \).