Math 307 Homework
September 14, 2015

1. Consider a linear system in matrix form:

$$\mathbf{A}\mathbf{x} = \mathbf{b},$$

   where $\mathbf{A} \in M_{m,n}(\mathbb{F})$ and $\mathbf{b} \in \mathbb{F}^m$ are given. Suppose that $\mathbf{x}, \mathbf{y} \in \mathbb{F}^n$ are both solutions. Under what conditions is $\mathbf{x} + \mathbf{y}$ also a solution?

   Remark: You might recognize this as a question we've encountered before. Don't just refer to the earlier answer; use what you've now learned about the matrix form of a linear system to answer the question more easily.

2. Find a 2×2 matrix with the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ as an eigenvector. Demonstrate that you are right, identify the eigenvalue, and explain how you found the matrix.

3. Let $\boldsymbol{P}_z : \mathbb{R}^3 \to \mathbb{R}^3$ be the function which orthogonally projects onto the x-y plane (that is, the set of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$ with $z = 0$). Is $\boldsymbol{P}_z$ linear? If so, prove it; if not, explain why not.