1. Consider the linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ whose matrix is

$$\begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}.$$ 

(a) Show that $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ are eigenvectors, and determine the corresponding eigenvalues.

(b) Draw the image of the unit square $\{(x, y) | 0 \leq x, y \leq 1\}$ under $T$.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the map defined by first rotating counterclockwise by $\theta$ and then reflecting across the line $y = x$. Find the matrix of $T$.

3. Define $T : C[0, \infty) \to C[0, \infty)$ by

$$Tf(x) = \int_0^x f(y) \, dy.$$ 

(Note that by the Fundamental Theorem of Calculus, $Tf$ is an antiderivative of $f$ with $Tf(0) = 0$.)

(a) Show that $T$ is linear.

(b) Show that $T$ is an integral operator (as discussed in class), although with a discontinuous kernel $k$. 

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