1. Determine which of the following are and are not subspaces of the given vector space, and justify your answers.

(a) The $x$ axis in $\mathbb{R}^3$.

(b) The set $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \geq 0 \right\}$ in $\mathbb{R}^2$ (i.e., the first quadrant of the plane).

(c) The set $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \geq 0 \right\} \cup \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \leq 0 \right\}$ in $\mathbb{R}^2$ (i.e., the first and third quadrants of the plane).

(d) The set of solutions of the linear system

\[
\begin{align*}
x - y + 2z &= 4 \\
2x - 5z &= -1.
\end{align*}
\]

2. (a) Show that $\mathbb{C}$ is a vector space over $\mathbb{R}$.

(b) Show that $\mathbb{Q}$ is not a vector space over $\mathbb{R}$.

3. The trace of an $n \times n$ matrix $A = [a_{ij}]_{1 \leq i \leq n}$ is

\[
\text{tr } A = \sum_{i=1}^{n} a_{ii}.
\]

Show that $S = \{ A \in \text{M}_n(\mathbb{F}) \mid \text{tr } A = 0 \}$ is a subspace of $\text{M}_n(\mathbb{F})$.

4. Let $V$ be a vector space, and suppose that $U$ and $W$ are both subspaces of $V$. Show that

\[
U \cap W := \{ v \mid v \in U \text{ and } v \in W \}
\]

is a subspace of $V$. 