Math 423 Extra problem 3 $\,$

Let $E\subseteq \mathbb{S}^{n-1}$ be a Borel set, and let

$$E_1 := \left\{ x \in \mathbb{R}^n \setminus \{0\} : |x| \in (0, 1], \frac{x}{|x|} \in E \right\}.$$

Show that $\sigma(E) := nm(E_1)$ is a Borel measure on \mathbb{S}^{n-1} .