1. Prove (more carefully than I did in class) that if $B(t)$ is a standard Brownian motion and τ is a stopping time, then $B(\tau)$ is $\mathcal{F}_\tau$-measurable.

2. Let $B(t)$ be a standard Brownian motion. Show that $B(t)^2 - t$ is a martingale.

3. Let $B(t)$ be a standard Brownian motion. Show that the following transformations of B are also Brownian motions (that is, they have independent centered Gaussian incremets with variance given by the length of the interval, and they have continuous sample paths):

   (a) $B_c(t) := \frac{1}{c}B(c^2t)$, where $c > 0$

   (b) $\tilde{B}(t) := \begin{cases} tB\left(\frac{1}{t}\right), & t > 0; \\ 0, & t = 0. \end{cases}$