# Math 492 Problem Set 1

1. Let $\{X_i\}_{i=1}^n$ and $\{X_i'\}_{i=1}^n$ be $2n$ i.i.d. random variables. Let $J$ be uniformly distributed in $\{1, \dots, n\}$. Show that if

$$W = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} X_j$$

and

$$W' = W - \frac{1}{\sqrt{n}} X_J + \frac{1}{\sqrt{n}} X_J',$$

then $(W, W')$ is an exchangeable pair.

2. Complete the proof of the Poincaré limit: let $X$ be a uniform random point on $\sqrt{n}S^{n-1}$ and let

$$W = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} X_j.$$

Use Stein's abstract normal approximation theorem to show that there is a constant $c$, independent of dimension, such that

$$d_{BL}(W, Z) \leq \frac{c}{\sqrt{n}}.$$