Fast model 5

Components
Plasma glucose – G, and stored glycogen: GL (liver), GM (muscle),
Plasma FFA – F, and stored triglycerides (adipose, liver) - T;
Byproducts of G- utilization (lactate, glycerols, pyruvate) – H
Amino Acids - A
Stored protein in skeletal muscle – P
Hormones: I – insulin, C – glucagon

Processes
• Source (food or injection): glucose, fat,
• Storage-release (conversion): G <-> GL; G→GM; G -> F<->T
• Utilization (uptake) of G, F (basal +brain + muscle);
• conversion of muscle byproducts into H
• Stimulation and utilization of hormones
• Basal levels and losses of G,F,I,C
Functions (Hill et al) for synthesis, uptake, control

Hormone controls: \[ SC = I \times C \]
\[ LC = I \times C \]
\[ SC = \phi_{m,e} \left( \frac{I}{I_0} \right) \cdot \psi \left( \frac{C}{C_1} \right) \]
\[ LC = \phi_{m,e} \left( \frac{C}{C_0} \right) \cdot \psi \left( \frac{I}{I_1} \right) \]

Substrate uptake/control (G,GL,GM,F)

Excess function
\[ Ef (z) = \frac{z - 1 + \sqrt{(z-1)^2 + \varepsilon^2}}{2} ; \text{ small } \varepsilon \]

Deficit function
\[ Df (z) = \frac{(1-z) + \sqrt{(z-1)^2 + \varepsilon^2}}{2} ; \]
Production rates

Rate of G/F uptake

\[ C_{G-GL} = \alpha_L \cdot \frac{SC}{\text{control}} \cdot \left(1 - \frac{GL}{GL_c}\right)_+ \cdot Ef\left(\frac{G}{G_1}\right) \]
\[ C_{G-GM} = \alpha_M \cdot \frac{SC}{\text{limited storage}} \cdot \left(1 - \frac{GM}{GM_c}\right)_+ \cdot Ef\left(\frac{G}{G_1}\right) \]
\[ C_{G-F} = \alpha_F \cdot \frac{SC}{\text{constrained storage}} \cdot \psi\left(\frac{F}{F_c}\right) \cdot Ef\left(\frac{G}{G_1}\right) \]
\[ \psi(z) = \frac{1}{1+z} \]
\[ C_{F-T} = \alpha_T \cdot \frac{SC}{\phi(F/F_0)}; \]
\[ C_{T-F} = \alpha_T \cdot \frac{SC}{\phi(T/T_0)}; \]

Rate of G/F production

\[ C_{GL-G} = \beta_L \cdot \frac{LC}{\phi(GL/GL_0)}; \]
\[ \phi(z) = \frac{z}{1+z} \]
\[ C_{H-G} = \beta_H \cdot \frac{LC}{\phi(H/H_0)}; \]

Muscle utilization

\[ C_{F-H} = v_M \cdot \frac{SC}{\phi(F/F_0)} = V_M \]
\[ C_{GM-H} = u_M \cdot \phi\left(\frac{GM}{GM_0}\right) = U_M \]

Parameters:

- \( GL_c, GM_c, F_c \) - max storage capacity (liver, muscle, FFA); \( G_1 \) – high G-threshold
- \( F_0, T_0, GL_0, GM_0, H_0 \) - low thresholds (FFA, triglycerides, liver, muscle, byproducts)
- \( \alpha_L; \alpha_M; \alpha_F; \alpha_T \) - max conversion rates (G -> “stored’)
- \( \beta_T; \beta_L; \beta_H \) - max extraction rates (‘stored’ -> G)
‘storage –release’ dynamics

\[
\begin{align*}
\frac{dG}{dt} &= S_G + C_{GL-G} + C_{H-G} - (C_{G-GL} + C_{G-GM} + C_{G-F}) - u_B \phi \left( \frac{G}{G_0} \right) - \mu_G G \\
\frac{dGL}{dt} &= C_{G-GL} - C_{GL-G} \\
\frac{dGM}{dt} &= C_{G-GM} - C_{GM-H} \\
\frac{dF}{dt} &= S_F + C_{G-F} + C_{T-F} - C_{F-T} - C_{F-H} - \mu_F F \\
\frac{dT}{dt} &= C_{F-T} - C_{T-F} \\
\frac{dH}{dt} &= C_{GM-H} + C_{F-H} - C_{H-G} - \mu_H H
\end{align*}
\]

- Decay rates

\[\mu_G; \mu_B; \mu_H; \mu_F\]

\[G_0\] - low G-threshold
Hormone DE (stimulation/loss)

\[
\frac{dI}{dt} = \mu_I (I_0 - I) + \gamma_I Ef \left( \left( \frac{G}{G_1} \right)^m \right) + \gamma_S \phi_1 \left( \frac{S_F + S_G}{S_0} \right) + \gamma_F Ef \left( \frac{F}{F_0} \right)
\]

basal production-loss  \quad excess-G stimulation  \quad 'food stimulation'  \quad excess-F stimulation

\[
- \nu_I \left( \frac{C_{G\rightarrow GL}}{\alpha_L} + \ldots + \frac{C_{GL\rightarrow G}}{\beta_L} + \ldots \right)
\]

I utilization

\[
\frac{dC}{dt} = \mu_C (C_0 - C) + \gamma_C Df \left( \left( \frac{G}{G_0} \right)^m \right) - \nu_C \left( \frac{C_{G\rightarrow GL}}{\alpha_L} + \ldots + \frac{C_{GL\rightarrow G}}{\beta_L} + \ldots \right)
\]

basal production-loss  \quad deficit-G stimulation  \quad I utilization

Parameters:

\( \gamma_I; \gamma_S; \gamma_C; \gamma_F \) - max stimulation rates of I by G, S (food), G-deficit, and F-excess

\( \mu_I; \mu_C \) - natural loss rates of I, C

\( \nu_I; \nu_C \) - max hormone utilization by G<-> ‘stored’ conversions

\( I_0; C_0 \) - basal levels of I,C ( = ? low I/C thresholds for controls SC, LC ?)

\( S_F; S_G \) - food source (fat, glucose)

\( S_0 \) - stimulation threshold
Fasting 48 hr
24-hour meal schedule:
8am-12pm-6pm
$S_G = 90$/meal;
$S_{Fat} = 50$/meal
Basal muscle utilization
3-day run:
8am-12pm-6pm
$S_G = 130/meal$;
$S_{Fat} = 80/meal$
Basal muscle utilization