1. We saw in class that for large $t$, every solution of

$$\frac{d^2y}{dt^2} + p \frac{dy}{dt} + qy = \cos \omega t$$

with $p, q > 0$ oscillates with angular frequency $\omega$ and amplitude

$$A = \frac{1}{\sqrt{(q - \omega^2)^2 + p^2\omega^2}}.$$

(a) What will the amplitude $A$ be like if $\omega$ is very small?

(b) Explain what your answer says about the effect of damping for certain frequencies of periodic forcing.
2. Consider the following model for two interacting species:

\[
\frac{dx}{dt} = 6x \left(1 - \frac{x}{3}\right) - xy,
\]

\[
\frac{dy}{dt} = 4y \left(1 - \frac{y}{4}\right) - xy.
\]

(a) What type of interaction do the two species have (predator-prey, cooperating, competing)? Justify your answer.

(b) Find the \(x\)-nullcline of the system.

(c) Find the \(y\)-nullcline of the system.
(d) Find all equilibria of the system.

(e) Classify each equilibrium \((x_0, y_0)\) with \(x_0, y_0 > 0\).
(f) Sketch the phase plane of the system for $x, y \geq 0$, indicating:

- Equilibria.
- Nullclines, including the directions of solution curves crossing the nullclines.
- Solution curves near each equilibrium $(x_0, y_0)$ with $x_0, y_0 > 0$.

(g) What is the long-term fate of the populations? Justify your answer.
3. Determine whether the following system is Hamiltonian, and if so, find a Hamiltonian function.

\[ \frac{dx}{dt} = x \sin y, \]
\[ \frac{dy}{dt} = \cos y. \]
4. Solve the initial value problem
\[
\frac{dy}{dt} = -y + 2u_3(t), \quad y(0) = 4.
\]
Formulae

\[ \mathcal{L}[y] = \int_0^\infty y(t)e^{-st} \, dt \]

\[ \mathcal{L}[y'] = s\mathcal{L}[y] - y(0) \]

\[ \mathcal{L}[y''] = s^2\mathcal{L}[y] - sy(0) - y'(0) \]

<table>
<thead>
<tr>
<th>(y(t))</th>
<th>(Y(s) = \mathcal{L}[y])</th>
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<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{s})</td>
</tr>
<tr>
<td>(e^{at})</td>
<td>(\frac{1}{s-a})</td>
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<tr>
<td>(u_a(t))</td>
<td>(\frac{e^{-sa}}{s})</td>
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<tr>
<td>(u_a(t)f(t-a))</td>
<td>(e^{-as}F(s))</td>
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