1. Suppose that $G$ is a finite group and $A < G$ is abelian. Prove that each irreducible representation (over $\mathbb{C}$) of $G$ has dimension at most $[G : A]$.

**Hint:** If $(V, \rho)$ is an irreducible representation of $G$, then $\rho_A$ defines a representation of $A$. For an irreducible subrepresentation $W$ of $A$, consider the subspaces $\rho(g)(W)$.

2. Let $(V_i, \rho_i)$ be representations of finite groups $G_i$ for $i = 1, 2$. The **tensor product** representation of $G_1 \times G_2$ is given by

$$\rho_1 \otimes \rho_2(g_1, g_2) = \rho_1(g_1) \otimes \rho_2(g_2) \in GL(V_1 \otimes V_2).$$

(a) Explain how this is related to, but different from, the tensor product of representations defined in class.

(b) Show that $\chi_{\rho_1 \otimes \rho_2}(g_1, g_2) = \chi_{\rho_1}(g_1) \chi_{\rho_2}(g_2)$.

3. Prove in detail that isomorphic representations have the same character.

4. Let $\chi_1, \ldots, \chi_N$ be the characters of the distinct (up to isomorphism) irreducible representations of $G$. Show that if $\chi$ is the character of any representation of $G$, then

$$\chi = \sum_{i=1}^{N} n_i \chi_i$$

for some integers $n_1, \ldots, n_N$. 