Here all groups are finite and all representations are finite dimensional representations over \( \mathbb{C} \).

1. Show that if \((V, \rho)\) is an irreducible representation of \( G \) and \((W, \sigma)\) is a 1-dimensional representation of \( G \), then \( V \otimes W \) is an irreducible representation of \( G \).

**Warning:** Make sure you notice this is referring to the tensor representation of just \( G \), so this is not a special case of problem 1 on the last homework.

2. (a) Determine the character table of the quaternion group \( Q_8 \).

(b) Give an example to show that two groups with the same character table need not be isomorphic.

3. Let \( G \) be abelian, and define \( \widehat{G} \) to be the set of all irreducible characters of \( G \). Show that \( \widehat{G} \) is an abelian group, with the binary operation given by pointwise multiplication.

4. Recall the (correct) character table for \( S_4 \):

\[
\begin{array}{c|ccccc}
& 1 & (12) & (123) & (1234) & (12)(34) \\
\hline
\tau & 1 & 1 & 1 & 1 & 1 \\
\varepsilon & 1 & -1 & 1 & -1 & 1 \\
\sigma & 2 & 0 & -1 & 0 & 2 \\
\psi & 3 & 1 & 0 & -1 & -1 \\
\varepsilon \otimes \psi & 3 & -1 & 0 & 1 & -1 \\
\end{array}
\]

Find the canonical decomposition of the representation \( \sigma \otimes \psi \).