1. Let $A \in M_{m,n}$. We saw that the pseudoinverse $A^\dagger$ satisfies:

(a) $AA^\dagger$ and $A^\dagger A$ are Hermitian;
(b) $AA^\dagger A = A$; and
(c) $A^\dagger AA^\dagger = A^\dagger$.

Show that these three properties above uniquely determine $A^\dagger$. That is, show that if $B \in M_{n,m}$ satisfies:

(a) $AB$ and $BA$ are Hermitian;
(b) $ABA = A$; and
(c) $BAB = B$,

then $B = A^\dagger$.

2. Given $A \in M_{m,n}$ and $b \in \mathbb{C}^m$, a least squares solution of the system $Ax = b$ is a vector $x \in \mathbb{C}^n$ such that $\|x\|$ is minimal among all vectors $x$ for which $\|Ax - b\|$ is minimal. Show that $x = A^\dagger b$ is the uniques least squares solution to $Ax = b$.

3. Prove that if $A \in M_n(\mathbb{F})$ is Hermitian, then there are an orthonormal basis $v_1, \ldots, v_n$ of $\mathbb{F}^n$ and numbers $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$ such that

$$A = \sum_{j=1}^{n} \lambda_j v_j v_j^*.$$