1. Let $A \in M_n$ and define $H(A) = \frac{1}{2}(A + A^*)$. Write the eigenvalues of $A$ as $\lambda_j = \alpha_j + i\beta_j$ for $\alpha_j, \beta_j \in \mathbb{R}$, and write the eigenvalues of $H(A)$ as $\mu_1, \ldots, \mu_n$. Show that $\alpha \prec \mu$.

2. Let $A, B \in M_{m,n}$. Show that

$$s(A + B) \prec s(A) + s(B),$$

where $s(\cdot)$ denotes the list of singular values of a matrix, in nonincreasing order. 

*Hint:* Use problems #3 and #4 from the February 4 homework.