1. Let $A, B \in M_n$, and let $1 \leq p, q \leq \infty$ with $\frac{1}{p} + \frac{1}{q} = 1$. Prove the noncommutative Hölder inequality:
\[
|\text{tr } AB| \leq \|A\|_p \|B\|_q.
\]

2. Let $A \in M_n$. Recall that $\frac{1}{2}(A + A^*)$ is called the Hermitian part of $A$. Prove that the Hermitian part of $A$ is the best Hermitian approximation of $A$ with respect to any self-adjoint norm (in particular, for any unitarily invariant norm). That is, prove that if $\|\cdot\|$ is self-adjoint, then
\[
\left\| A - \frac{1}{2}(A + A^*) \right\| \leq \|A - H\|
\]
for every $H \in H_n$. 