1. Suppose that $G = (V, E)$ is a connected (undirected) graph; that is, there is a path along edges from any edge $i$ to any other edge $j$.

(a) Show that the adjacency matrix $A$ and the transition matrix $P$ for the random walk on $G$ are irreducible.

(b) Show that the stationary distribution $\pi$ given by $\pi_i = \frac{d_i}{2|E|}$ is the unique stationary distribution for $P$.

(c) Show that for any $0 < \varepsilon < 1$, the matrix $Q = (1 - \varepsilon)P + \varepsilon I_n$ is an irreducible, primitive, stochastic matrix.

(d) Show that $e^T Q^m \to \pi$ as $m \to \infty$.

(e) Suppose that the total number of edges $|E|$ in some graph is unknown, but for a given vertex you can tell its degree. Explain how you could use random walk on the graph to estimate $|E|$. (This is a very simple example of Markov Chain Monte Carlo techniques.)