

Errata to *Alice and Bob Meet Banach: The Interface of Asymptotic Geometric Analysis and Quantum Information Theory* by Guillaume Aubrun & Stanisław J. Szarek

Last updated on September 7, 2023

We – and undoubtedly other readers – will greatly appreciate bringing the typos/errors to our attention; you may use the email links at <http://www.ams.org/bookpages/surv-223>. We also welcome more substantive comments (improvements, solutions to problems, and the like), either by email or via comments to the blog at <https://aliceandbobmeetbanach.wordpress.com/>.

As a rule, the replaced or deleted text (if applicable) is marked below in red, while the new (correct) text is in blue. For straightforward typos, we write **string1** → **string2** to indicate that the text “string1” is to be replaced by “string2.”

P. 12, inequality (1.4): $n^{1/p-1/q}\|x\|_p \rightarrow n^{1/p-1/q}\|x\|_q$

P. 23, line 1: $y = Bx \rightarrow x = By$

P. 24, line –11: $M_n \rightarrow M_m$

P. 24, line –3: $\overline{u_{ji}} \rightarrow \overline{u_{ij}}$

P. 37, the first line of Definition 2.8: $\rho = |\chi\rangle\langle\chi|$ on $\mathcal{H} \rightarrow \rho$ on \mathcal{H}

P. 37, line –9: **linear** → **linearly**

P. 79, Exercise 4.1: **Let $H \subset \mathbb{R}^n$ be a hyperplane** → **Let $H \subset \mathbb{R}^n$ be a linear hyperplane**

P. 80, Exercise 4.2 should read as follows.

Let $K \subset \mathbb{R}^n$ be a convex body and let Δ be the simplex of largest volume contained in K . Show that if 0 is the centroid of Δ , then $K \subset -n\Delta \subset n^2\Delta$ and $K \subset (n+2)\Delta$. In particular, if Δ_n is the regular n -dimensional simplex, then $d_{BM}(K, \Delta_n) \leq n+2$.

Two occurrences of $n+1$ in the last two lines were replaced by $n+2$, this is what the argument suggested in the hint on p. 339 actually gives. In the inclusion $K \subset (n+2)\Delta$, the factor $n+2$ is optimal. (The reference for these is [M. Lassak, Approximation of convex bodies by inscribed simplices of maximum volume. Beitr. Algebra Geom. 52 (2011), no. 2, 389-394].)

P. 103, Notes and Remarks on Section 4.1: We incorrectly cite a result from [GLMP04]; it should read “ $d_{BM}(K, \Delta_n) \leq n$ for any **centrally symmetric** convex body $K \subset \mathbb{R}^n$.” Under this symmetry assumption and in this generality, this actually follows from Exercise 4.2 (and in fact is an equality; [GLMP04] asserts further that $d_{BM}(K, L) \leq n$ if one of the bodies K, L is centrally symmetric). Indeed, $K \subset -n\Delta$ implies then that K is contained in some translate of $n\Delta$, which is therefore a homothetic image of Δ – with ratio n – with respect to some center (recall that, by construction, $\Delta \subset K$).

Since the center of symmetry of K may be different from the centroid of Δ (assumed to be 0), the location of the center of homothety is not immediately clear from this argument. For example, in [GLMP04] examples are cited with the center belonging to the boundary of Δ , which is not ideal for some applications. It is not completely clear what the optimal factor is if we accept any simplex (i.e., not necessary one of largest volume), but still insist that the center of the homothety is its centroid.

For not-necessarily-symmetric bodies $K \subset \mathbb{R}^n$ it appears to be known that, at least in some cases, we may have $d_{BM}(K, \Delta_n) > n$. For example, in [R. Fleischer, K. Mehlhorn, G. Rote, E. Welzl and C. Yap, Simultaneous inner and outer approximation of shapes. Algorithmica 8 (1992), 365-389] it is asserted that the distance between a triangle and a regular pentagon equals

$R := 1 + \frac{1}{2}\sqrt{5} \approx 2.118$ and conjectured that *always* $d_{BM}(K, \Delta_2) \leq R$. (What is shown is that $d_{BM}(K, \Delta_2) \leq 9/4$. See also [M. Lassak, Approximation of convex bodies by inscribed simplices of maximum volume. Beitr. Algebra Geom. 52 (2011), no. 2, 389-394].) It is apparently believed that R is in fact the largest possible distance between two 2-dimensional convex bodies, though we couldn't identify the author of this conjecture.

P. 182, the first paragraph of Section 7.1.2: The range of the projection R_d does not really consist of homogeneous polynomials of degree d , but rather is the span of the Hermite polynomials of total degree d . (While the definition (5.56) does *look* homogeneous, the point is that univariate Hermite polynomials of degree larger than 1 are not homogeneous.)

P. 215, the paragraph containing (8.3): $-2 \log \max \lambda_1 \rightarrow -2 \log \lambda_1$

P. 228, Proposition 8.24: $\Phi : M_m \rightarrow M_d$ needs to be replaced by $\Phi : M_m \rightarrow M_k$ and, consequently, $V : \mathbb{C}^m \rightarrow \mathbb{C}^d \otimes \mathbb{C}^d$ needs to be replaced by $V : \mathbb{C}^m \rightarrow \mathbb{C}^k \otimes \mathbb{C}^d$. The proof does not need to be modified.

P. 234, the last paragraph of the Notes and Remarks on Section 8.5:
 $g_{\min}((\mathbb{R}^3)^{\otimes 4}) = 1/\sqrt{7}$ needs to be replaced by $g_{\min}((\mathbb{R}^3)^{\otimes 3}) = 1/\sqrt{7}$.

P. 248, lines 3-4: $\mathbf{E}[\theta_j \bar{\theta}_k \theta_l \bar{\theta}_m]$ \rightarrow $\mathbf{E}[\theta_j \bar{\theta}_k \bar{\theta}_l \theta_m]$ and, consequently, in lines 4-5:
 (2) $j = m$ and $k = l \rightarrow$ (2) $j = l$ and $k = m$.

P. 339, Hint to Exercise 4.2: The factor $(n + 1)$ in the second line should be replaced by $(n + 2)$, see the correction to the statement of the Exercise on p. 80.

P. 340, Hint to Exercise 4.3: The factor $(n + 1)$ in the third line should be replaced by $(n + 2)$, see the correction to the statement of Exercise 4.2 on p. 80.

We thank the following colleagues for pointing out errors (or otherwise contributing to the above):

Marek Lassak

Rupert Levene

Adam Lutoborski

Mark Meckes

Alexander Müller-Hermes

Timur Oikhberg