The objective is to use triple integrals to calculate the mass and the centroid of a solid that is described geometrically. The solid is not homogeneous, its density varies from point to point and is described by a formula. You may work on this assignments in pairs. Ideally, the assignment should be submitted via email as a Mathematica notebook. The more onerous outputs can (and sometimes should) be suppressed by typing “;” (a semi-colon) at the end of the corresponding command.

Consider first the disk \((x - 5)^2 + z^2 \leq 9\) in the \(xz\)-plane and the \textit{solid torus} obtained by revolving this disk around the \(z\)-axis. Our solid is the part of the torus that lies \textit{above} the paraboloid \(z = 1 - (x^2 + y^2)/4\) and further satisfies \(x \geq 0, y \geq 0\). The density of mass \(\rho\) is given by the formula \(\rho(x, y, z) = (x + 2y^2) \ast (z + 3)\).

While it is possible to give the exact expression for the quantities in question, it is not practical as the answer would take several pages. Accordingly, the answers are to be given as decimals rounded to the nearest 1/1000th (i.e., 3 digits after the decimal point).

\textit{Hints:}  \(1\) One possible approach is to express the equations and evaluate the integrals in cylindrical coordinates. \(2\) Splitting the region of integration into subregions may be necessary, but once the splitting and the limits of integration are worked out, they may be used for evaluating all integrals. \(3\) Integrate the iterated integral step by step, simplifying the answer after each integration. \(4\) To visualize the region, it may be helpful to plot its intersection with (say) the \(xz\)-plane. To find some important points it may be necessary to use the function \texttt{Solve}. \(5\) Some information about using Mathematica in general and on the CWRU Network in particular may be found at Dr. Hurley’s Web page at http://www.case.edu/artsci/math/hurley/