MATH 322/422 Spring 2022

Assignment #1, Extra problem

Let $f : [a, b] \to \mathbb{R}$ be a convex function. Show that f is bounded on $[a, b]$.

Note & Hint : Remember that, as was pointed out in class, it follows that f is continuous on the open interval (a, b), but not necessarily on $[a, b]$. One possible approach is to apply the same trick ("squeezing" between two linear functions) that was used in class to prove continuity on (a, b).

More notes : For the purpose of this assignment, you are allowed to use the elementary properties of the trigonometric and exponential functions (including the formulae for their derivatives), even though we haven't rigorously defined such functions yet. In section 7.11 problems, do not forget about verifying the hypotheses of the theorems you are using, that will be worth half the credit. The verifications do not need to be detailed, but the properties that are being used must be stated clearly. For example, when considering $\lim_{x\to 0} \frac{1-\cos x}{x^3}$ (an example studied in class) an explanation *"This is a* $0/0$ *indeterminate form, so we can attempt l'Hôpital's Rule"* is sufficient. An excellent argument starts with *"This is a* $0/0$ *indeterminate form; given that* $f(x) = 1-\cos x$ *and* $g(x) = x^3$ *are continuous functions, their limits at* 0 *are found by evaluating* $f(0)$ *and* $g(0)$*."*