MATH 322/422 Spring 2022

Assignment #3, extra problems and notes

Problem A Suppose $f, g \in \mathcal{R}[a,b]$ are such that $f \leq g$ on $[a,b]$. Show that $\int_a^b f \leq \int_a^b g$ using the approach via upper/lower sums.

Problem B (a) Suppose that $f : [a,b] \to \mathbb{R}$ is continuous (hence $f \in \mathcal{R}[a,b]$) and that $f \geq 0$ on $[a,b]$. It then follows from Problem A that $\int_a^b f \geq 0$. Show that if $\int_a^b f = 0$, then $f(x) = 0$ for all $x \in [a,b]$.
(b) Show that the statement above is false if we drop the continuity hypothesis (i.e., give an appropriate counterexample and briefly justify why it works).

Notes and hints :

Problem A is Theorem 8.24 from the text. You are allowed to use the properties shown in class and/or included in the notes posted on Canvas. Reproducing the argument given in the text will not be accepted.

Hint for Problem B(a) : Argue by contradiction, suppose that there is $c \in [a,b]$ such that $0 < f(c) =: \alpha$, and deduce that there is a subinterval of $[a,b]$ on which f is bounded from below by $\alpha/2$.

Exercise 8.2.8 Use the method suggested in Exercise 8.2.7 specified to the context of Exercise 8.2.8. You will need to justify the steps. Choose tags in such a way that you can calculate the Riemann sums (note the mention of 'geometric progressions'...)

Exercise 8.2.11 Interpret the expression in the problem as a Riemann sum (or an upper/lower sum) of some function on the appropriate interval. You are allowed to use "the usual calculus method" to calculate the integral, but you will need to rigorously justify convergence.