**MATH 322/422** Spring 2022

**Assignment #4, notes and hints**

*Problem 3.6.21* This is pretty straightforward from the proof in Section 3.6.8 (or its version shown in class). The last question asks for the “best” two-sided inequalities relating the two quantities (i.e., which one is larger and by how much in the extreme).

*Problem 8.3.6* This is based on the same ideas as Extra Problem B from Assignment #3 (solutions will be posted on Canvas lated today).

*Problem 8.4.7* For the improper integral over $[-1, 1]$ to exist, both integrals $\int_{-1} 0$ and $\int_0^1$ must converge (as improper integrals). This needs to be shown if the answert is YES. If the answer is NO, you need to provide a counterexample and justify. *Hint* : If $\lim_{x\to c}(g(x) + h(x))$ exists, does it follow that $\lim_{x\to c} g(x)$ and $\lim_{x\to c} h(x)$ exist?

*Problems 8.5.6, 8.5.9* : The most important hypothesis in the Integral Test 3.35 is the function $f$ being nonincreasing.

*Exercise 8.6.2* There is a simple proof using one of the advanced theorems about Riemann integrability, or one can argue via lower/upper sums (this will be a somewhat tedious even if not very hard calculation). In both cases you will have to use the fact that Riemann integrable functions are bounded.

*Exercise 8.7.3* All parts of this problem are easy except for question (b), which will be counted for extra credit. A more “to the point” version of that question is: *“If $x$ is a point for which $\lim_{t\to x} f(t)$ doesn’t exist, is it still possible that $F'(x)$ exists?”*