MATH 322/422 Spring 2022

Assignment #5, notes and hints

Problem 9.2.1 "Examine the pointwise limiting behavior" means determining (for each $x \in \mathbb{R}$) whether $(f_n(x))$ converges or not, and finding the limit. Equivalently, finding $D = \{x \in \mathbb{R} : (f_n(x)) \text{ converges}\}$ and the limit function $f : D \to \mathbb{R}$.

Problem 9.2.3 It is not necessary to justify all claims "from scratch." For some of them, it will be enough to refer to an example or a statement from the text and/or presented in class.

Problem 9.2.7 Answer TRUE or FALSE and, as appropriate, state a concise reason why true, or give a counterexample showing why not true. You do not have to explain why the counterexample works, as long as it does work. Either way, the answer to each part should fit in two lines. Note that only parts (a)-(e) are assigned. Remember that by "increasing" the text means "strictly increasing."

Problem 9.3.1 The limiting function was found in Problem 9.2.1. You are expected to (potentially) answer two queries, with justification:
(i) Is the sequence (f_n) uniformly convergent on the entire set D of pointwise convergence (likewise found in Problem 9.2.1)?
(ii) If the answer to question (i) is NO, give examples of interesting subsets of D on which the convergence is uniform, in the spirit of the following observation: *The sequence $f_n(x) = x^n$ converges pointwise on $[0, 1]$. While the convergence is not uniform on $[0, 1]$ (as shown in class), it is uniform on each interval $[0, r]$ for $r < 1$.*

Problem 9.3.8 is NOT assigned (it was included in the tentative assignment that was posted earlier).