Citations

Page number(s) in parentheses show the pages of this Handbook on which the quotation is referenced.

[p. 827. Lines 5–7.]
Let \( c = (c_0, \ldots, c_s) \in \mathbb{R}^{s+1} \) be a finite sequence of real numbers. If \( c_0 \ldots c_s \neq 0 \), the number of variations in sign of \( c \) is the number of indices \( 1 \leq j \leq s \) such that \( c_{j-1}c_j < 0 \) (that is, such that \( c_{j-1} \) and \( c_j \) have opposite signs.)

[p. 806. Lines 20–22, second column.]
Now taking generators for $IG$ as a $ZG$-module, we can map a free $ZG$-module of finite rank onto $IG$. 

[p. 619. Lines 12–13.]

**Definitions.** By an $N$-cube of order $T$, we mean an $N$-dimensional array of $T^N$ $N$-dimensional boxes.

[p. 813. Lines 12–13.]

Now, we associate to each $i \in V$ a variable $x_i$ (over $\mathbb{R}$) and consider the function $f(x_1, \ldots, x_n) = 2 \sum_{ij \in E} x_i x_j$. 

[p. 243. Line 3.]
We have already proved (a) and the formal proof of (b) is very similar.
Citations


[p. 232. Lines 4–2 from bottom.]

How do you actually draw a 2000-edge tree . . . which “looks right”, and what metric space do such trees inhabit?

[p. 333. Lines 3–2 above Figure 1.]

By an inductive argument, all \((n - 1)!\) arrangements of the lower cards are equally likely.

Substituting into the above formula, we obtain

\[5x^2 + 20x + 30 = z^2.\]

Since 5 is a factor of all terms on the left, it must be a factor of z.

[p. 16. Proposition 3 and lines 6 and 11–12 of the proof.]

Proposition 3. If $G$ is a $QD$-group with quasi-dihedral Sylow 2-subgroups whose unique nonsolvable composition factor satisfies the First Main Theorem, then for any involution $x$ of $G$ and any Sylow 2-subgroup of $S$ of $C(x)$, we have $[O(C(x))] \subseteq O(G)$. . . . Let $L$ be the simple normal $QD$-subgroup of $G$. . . . Clearly $L \cap O(N) = L \cap O(C(x)) \subseteq O(C_1(x))$. But by assumption $L$ satisfies the conclusion of the First Main Theorem . . .
Let $p(x)$ be a polynomial of degree $n$ with all real distinct roots $x_1 < x_2 < \cdots < x_n$. Suppose we “drag to the right” some or all of these roots. I.e. we construct a new $n$th degree polynomial $q$ with all real distinct roots $x'_1 < x'_2 < \cdots < x'_n$ such that $x'_i > x_i$ for all integers $i$ between 1 and $n$. The derivatives of $p$ and $q$, which of course are polynomials of degree $n - 1$, must also have all real distinct roots from Rolle’s theorem.
Let us set
\[ \eta(x, t) = \phi(x)\psi(t)e \]

_Proof_. By hypothesis, the coefficients $a$, $b$, and $c$ are not all zero. Assume, for the moment, that $a \neq 0$. Then the equation $ax + by + cz + d = 0$ can be rewritten as $a(x + (d/a)) + by + cz = 0$. But this is a point-normal form of the plane passing through the point $(-d/a, 0, 0)$ and having $n = (a, b, c)$ as a normal.

Using Theorem 3.9 we express $\det A$ in terms of its $k$th-row cofactors by the formula

$$ (3.30) \quad \det A = \sum_{j=1}^{n} a_{kj} \cof a_{kj} $$

Keep $k$ fixed and apply this relation to a new matrix $B$ whose $i$th row is equal to the $k$th row of $A$ for some $i \neq k$, and whose remaining rows are the same as those of $A$. Then $\det B = 0$ because the $i$th and $k$th rows of $B$ are equal. Expressing $\det B$ in terms of its $i$th-row cofactors we have

$$ (3.31) \quad \det B = \sum_{i=1}^{n} b_{ij} \cof b_{ij} = 0. $$

[p. 56. Lines 9–8 from the bottom.]

**Definition.** The ω-group $G_1$ is ω-homomorphic to the ω-group $G_2$ [written: $G_1 \geq \omega G_2$] if there is at least one ω-homomorphism from $G_1$ onto $G_2$. 

[p. 169. Lines 1–4.]

A group may be thought of as a set with 3 operators, a binary operation labeled $\cdot$ (we say the label $\cdot$ has *arity* 2 since $\cdot$ labels a 2-ary operator); a unary operation labeled $^1$ (which has arity 1), and a constant labeled $e$ (we say $e$ has arity 0, and refer to constants as nullary operators).

[p. 464. Lines 6–5 from bottom.]

Now by the preceding paragraph we can find an $m$-null set $M \subseteq X$ such that, for all $x \in M$, $|\mu_x|$ lives in $G_x$. 

16
Suppose now we move on to an equation with two unknowns, for example

\[ x^2 + y^2 = 1, \quad (1) \]
\[ x^3 + y^2 = 1. \quad (2) \]

[p. Abstract. Third through fifth sentences.]

Here we report on a particular approach to providing such facilities, called “orthogonal persistence”. Persistence allows data to have lifetimes that vary from transient to (the best approximation we can achieve to) indefinite. It is orthogonal persistence if the available lifetimes are the same for all kinds of data.

[p. 142. Proposition 3.4.]
The generalized eigenvectors of $T$ span $V$. 

[p. 153. Lines 8–7 from bottom.]

Thus the function $A$ defined on $\text{ran}\sqrt{S^*S}$ by $A(\sqrt{S^*S}u) = Su$ is well defined and is a linear isometry from $\text{ran}\sqrt{S^*S}$ onto $\text{ran}S$. 

[p. 16. Lines 18–19.]

Definition. The group $G$ is supersoluble if every homomorphic image $H \neq 1$ of $G$ contains a cyclic normal subgroup different from 1.

[p. 835. Lines 4–3 above Figure 2.]
In Fig. 2(a) below, a nine-point graph is shown.
Citations


[p. 451. Theorem 1.]

An apportionment method $M$ is house-monotone and consistent if and only if it is a Huntington method.

[p. 641. Lines 11–8.]

We call \( \phi : \mathbb{R}^n \to \mathbb{R} \) a Lipschitz function if there exists a constant \( M \) such that \( |\phi(x) - \phi(y)| \leq M |x - y| \) for all \( x, y \in \mathbb{R}^n \). The smallest such \( M \) for which this inequality remains valid for all \( x \) and \( y \) will be called the Lipschitz constant of \( \phi \).

[p. 429. Lines 10–11.]

We follow the customary approach in elementary calculus courses by using the definition \( \ln(x) = \int_1^x t^{-1} \, dt \).

[p. 626. Lines 17–19 and 36.]

Usually the partition is ordered and the intervals are specified by their end points; thus \( I_i := [x_{i-1}, x_i] \), where

\[
a = x_0 < x_1 < \cdots < x_{i-1} < x_i < \cdots < x_n = b.
\]

\( \ldots \) A strictly positive function \( \delta \) on \( I \) is called a *gauge* on \( I \).

[p. 627. Lines 23–24.]

**(3.2)** If $h : [0, 1] \to \mathbb{R}$ is Dirichlet’s function (= the characteristic function of the rational numbers in $[0, 1]$), then $h \in \mathcal{R}^*[0, 1]$ and $\int_0^1 h = 0$. 

[8.2] Dominated Convergence Theorem. Let \((f_n)\) be a sequence in \(\mathcal{R}^*([a, b])\), let \(g, h \in \mathcal{R}^*([a, b])\) be such that

\[
g(x) \leq f_n(x) \leq h(x) \quad \text{for all} \quad x \in [a, b],
\]

and let \(f(x) = \lim_n f_n(x) \in \mathcal{R}\) for all \(x \in [a, b]\). Then \(f \in \mathcal{R}^*([a, b])\) and (8a) holds.

\[
\text{... As usual, we define a null set in } I := [a, b] \text{ to be a set that can be covered by a countable union of intervals with arbitrarily small total length.}
\]

\[
\text{... Every } f \in \mathcal{R}^*(I) \text{ is measurable on } I.
\]

[p. 845. Lines 22–23.]

We put $i(e) = [\mathcal{A}_0, \alpha_e \mathcal{A}_e]$ and call $(A, i) = I(A)$ the corresponding *edge-indexed graph*. 


The theorem is an asymptotic formula for the counting functions of primes $\pi(x) := \# \{ p \leq x : p \text{ prime} \}$ asserting that

\[ \pi(x) \sim \frac{x}{\log x} \]

The twiddle notation is shorthand for the statement

\[ \lim_{x \to \infty} \frac{\pi(x)}{\{x/\log x\}} = 1. \]

[p. 738. Lines 12–10 from bottom.]

In 1937 A. Beurling introduced an abstraction of prime number theory in which multiplicative structure was preserved but the additive structure of integers was dropped.

[p. 318. Just below the first figure.]

The formula in parenthesis-free (polish) form [2] is now written over the fixed bars . . .

[p. 644. Lines 4–2 above the figure.]

We obtain

$$\Phi(X) = \{(x, u(x)) \mid x \in [a, b]\}$$

which is the graph of the function $u$. 

[p. 3902. Lines 20–24.]

Therefore we can assume w.l.o.g. that \( r^i_{s_i t_i} = 0 \) for \( 2 \leq i \leq l \). Using the linear independence of \( \Phi_1, \ldots, \Phi_l \) over \( \wedge^2 A \) we iterate this procedure. Therefore we can assume w.l.o.g. that there are pairwise distinct pairs \((s_i, t_i)\) such that \( r^i_{s_i t_i} \neq 0 \) and \( r^j_{s_i t_i} = 0 \) for \( i \neq j \).

Our starting point is the well known identity
\[ 1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2 \]

[p. 705. Lines 12–14.]

In fact it turns out that any nontrivial admissible $F_2$ (that is, $F_2 \in C^1[0,1]$ with $F_2(0) = F_2(1) = 0$ but $F_2(x) \not\equiv 0$) satisfies the second of the above inequalities.
Citations


[p. 332. Lines 3–1 from bottom.]

The idea of the proof is to unwind the proof of $MS_1$ given in §1.3 into a direct diagonal construction, and modify that construction so as to obtain the extra information needed to realize the conclusion from the realizing functionals of the hypothesis.

[p. 19. Lines 8–10.]

It is quite simple to reduce the problem to showing the existence of a certain “pathological” function defined (not necessarily on all reals but) on indices provable in $R$. 

A continuum is a compact metric space. $I = [0, 1]; A = (0, 1]; S$ is the unit circle in the complex numbers. If $X$ is a continuum, a pseudocone over $X$ is a compactification of $A$ with remainder $X$. 
Proof. Let $N$ be the set of those positive integers which satisfy the following conditions:
(a) 1 is a member of $N$,
(b) whenever $x$ is a member of $N$, then $x \geq 1$.
We need to show that $N$ is precisely the set of all positive integers to prove our result.

Throughout we let $f$ denote a real valued function defined on the real line $\mathbb{R}$. 
Complete the square in (3.2) to obtain an equation of the form
\[(F + \text{stuff})^2 = G(x, y, z, F_1).\] (3.3)
Let \(z = Z(x, y)\) stand for the value of \(z\) for which the left side of (3.3) vanishes. Since the left side of (3.3) is a square, its derivative with respect to \(z\) also vanishes at \(Z\). Applying this to the right side of (3.3) we obtain the two equations
\[G(x, y, Z, F_1) = 0\] and \[G_z(x, y, Z, F_1) = 0\]
in the two unknowns \(Z\) and \(F_1\). These are rational equations, and they can be “solved” for \(F_1\).
The Toda bracket \( \langle - , 3 , \alpha_1 \rangle \) is essentially multiplication by \( v_1 \), which acts nontrivially from \( v_{4i+2}S^{23} \) to \( v_{4i+6}S^{23} \).

[p. 271. Lines 8–6 from bottom.]

We shall occasionally use lambda-notation to specify functionals, i.e. \( \lambda X. \ldots \) specifies a functional \( F \) such that \( FX = \ldots \) for all \( X \).

[p. 502. Lemma 3.]

**Lemma 3.** Let $f$ and $g$ be as in Lemma 2, and let $u \in E_n$ be a unit vector. Then for every $x \in E_n$ orthogonal to $u$ we have $|\langle f(x), g(u) \rangle| \leq 3\varepsilon$. 

[p. 503. Line 9 from bottom.]

Since \( f \) is an \( \varepsilon \)-isometry, we have

\[ m - \varepsilon < \| f(x + my) - f(x) \| < m + \varepsilon, \]

or equivalently,

\[ m - \varepsilon < \| (m - a + b_m)y + u_m \| < m + \varepsilon. \]

[p. 425. Abstract.]

We prove that $\Delta(G)$ satisfies a certain rigidity property and apply this to give a new and conceptual proof of the Brewster-Roseblade result [4] on the group of automorphisms stabilizing $G$. 
The singular solution of Clairaut’s equation is the envelope of this family of lines and can be parametrized by the pair of equations $x = -f'(t)$ and $y = -tf'(t) + f(t)$. On the other hand, we could consider a one-parameter family $\mathcal{F}$ of lines $y = f(b)x + b$ in which the parameter is the $y$-intercept of each line.
For an illustration of this theorem, suppose the $A$ in (10) is the set $[x : \alpha \leq x(1) \leq \beta]$ of paths in $C_0[0, 1]$ that over the point $t = 1$ have a height between $\alpha$ and $\beta$. 


[p. 1107. Lines 3–2 from bottom.]

[p. 389. Lines 10–12.]

In both (46) and (47) it is to be noted that the Σ-terms are not important except at $x = a$ and $x = b$, since the real parts of the exponential terms are large and negative for $l$ large and $a < x < b$. 

[p. 279. Lines 23–24.]

Using Lemma 2, we find a closed braid representative $K_\beta$ of $B_K$, $\beta \in B_n$. 

[p. 96. Lines 12–13 and 15.]

**Theorem 4.** Let $S$ denote the boundary point of the ordinary set. Then $S$ is either on the screen or on the envelope of the family of separation lines.

*Proof.* ... Suppose $S$ is a boundary point of the set of ordinary points that does not belong to the screen.
[p. 122. Lines 19–20.]
For any two $G$-sets $X$ and $Y$, there are obvious actions of $G$ on the disjoint union $X + Y$ and (componentwise) on the product $X \times Y$. 

Citations


[p. 529. Lines 6–7 of Introduction.]

A 3-smooth number is a positive integer whose prime divisors are only 2 or 3.
**Corollary.** Assume that $n > 1$ is prime to 6. Then $n$ has a unique representation if and only if the 2-rep and the 3-rep of $n$ agree.

[p. 915. Lines 10–8 from bottom.]

In order to give a quick treatment for the $n$ by $n$ case, it is tempting to give a known formula for the determinant, such as the permutation or the column expansion formula, . . .
The basic tool for determining the local behavior of a function at a point of its domain is Taylor’s theorem. The problem whose solution we pursue is the following: Given a point \( a \) in the domain of a sufficiently smooth function \( f \), what is the least degree of a Taylor polynomial of \( f \) at \( a \) that adequately describes the local behavior of \( f \) at \( a \)?

[p. 41. Lines 4–2.]

So, for example, no Carmichael number has the prime factors 3 and 7 at the same time. This property was used by Giuga to prove computationally that each counterexample has at least 1000 digits.
Citations


[p. 204. Lines 12–15.]

In part we perhaps settle for computing digits of $\pi$ because there is little else we can currently do. We would be amiss, however, if we did not emphasize that the extended precision calculation of pi has substantial application as a test of the ”global integrity ” of a supercomputer.

[p. 717. Lines 19–21.]

But the analyticity of $G(z, w)$ requires that the numerator of the fraction vanish whenever the denominator does.
The stated conditions imply that the mass of the vehicle is given by
\[ m(t) = m_0 - \beta t \quad t < t_1 \]
At the time \( t_1 \), the mass of the vehicle has been reduced to the value \( m_1 = m_0 - \beta t_1 = m_0 - m_f \).
The Poincaré polynomial of $M$ is $P_M(t) = \sum b_i t^i$ where $b_i$ is the $i$th Betti number of $M$. Thus rank $M \geq 1$ iff $-1$ is a root of $P_M(t)$. 


[p. 592. Lines 3–5.]

The Poincaré polynomial of $M$ is $P_M(t) = \sum b_i t^i$ where $b_i$ is the $i$th Betti number of $M$. Thus rank $M \geq 1$ iff $-1$ is a root of $P_M(t)$. 

63
We also consider the trivial space \{0\} to be of finite dimension 0.
Citations


[p. 184. Lines 7–9.]

The mapping behaves very differently at singular points from the way it does at regular ones, where locally it behaves as if it were a mapping onto its codomain.
Lemma 2. For each circle or extended line $E$, there is a unique $\bar{T} \in \bar{\mathcal{M}}$ such that

$$E = \{ z \in \hat{\mathbb{C}} : \bar{T}(z) = z \}.$$ 

(E is exactly the set of fixed points of $\bar{T}$.) This $\bar{T}$ is an involution of $\hat{\mathbb{C}}$; that is, $\bar{T} \circ \bar{T}$ is the identity.
We get the inequality we want as long as
\[
\frac{\sqrt{6n} + 2^{k+1}}{\sqrt{n} + 2^{2k+1}} < \sqrt{6},
\]
i.e., as long as \(\sqrt{6n} + 2^{k+1} < \sqrt{6n + 3 \cdot 2^{2k+2}}\). The latter inequality is equivalent to \(n < 2^{2k+1}/3\), which is true in the interval we’re considering (\(n \leq M_{k-1}\)).

Let $B_n$ be the algebra which has as the base set all $n$-square Boolean matrices and is endowed with the component-wise Boolean operations of complementation $'$, meet $\cdot$ and join $+$ (under which it forms a Boolean algebra) as well as the matrix operations of transposition and multiplication $;,$ and the identity matrix $I$. 

[p. 370. Theorem 1.]

Theorem 1 (LUCAS-LEHMER). Let $p$ be a prime number. Then $M_p = 2^p - 1$ is a prime if $M_p$ divides $S_{p-1}$. 

[p. 140. Lines 2–3.]

As an illustration of this theorem let us consider a function $u$ with the following properties . . .
**Theorem**  For any integer $n > 1$ there exist at most $n - 1$ mutually orthogonal $n \times n$ Latin squares.

[p. 60. Lines 1–3.]

Let $R$ and $K$ be subspaces of a Hilbert space $H$, and let $P_R$ and $P_K$ denote the orthogonal projections of $H$ onto these subspaces. When is the operator $P_R - P_K$ invertible? . . .

[p. 513. Lines 8–9.]

Which finite sequences correspond to juggling patterns? Certainly a necessary condition is that the average must be an integer. However this isn’t sufficient.

[p. 796. Problem 10331.]

Find all positive integers $n$ such that $n!$ is multiply perfect; i.e., a divisor of the sum of its positive divisors.
   [p. 499. Problem 10311.]

It is well-known that if $g$ is a primitive root modulo $p$, where $p > 2$ is prime, either $g$ or $g + p$ (or both) is a primitive root modulo $p^2$ (indeed modulo $p^k$ for all $k \geq 1$.)

[p. 1129. Lines 4–6.]

It seems that none of the textbooks on linear algebra gives a direct proof of the fact that the minimal polynomial $m(x)$ of a linear transformation $T$ is of degree less than or equal to the dimension of the vector space $V$ on which $T$ acts. The usual proof depends on the fact that $m(x)$ divides the characteristic polynomial $f(x)$, the degree of which equals the dimension of $V$, and for this one needs the Cayley-Hamilton Theorem.
The new algorithm can now be described. To approximate roots of $P(x) = 0$ (which, without loss of generality, is assumed not to have a root at $x = 0$):

Step 1: If the desired root is known to be near the origin, solve $P(1/z) = 0$ for $z = 1/x$.

Step 2: Determine a preliminary root estimate $R \neq 0$.

Step 3: Use $R$ and formula (6) to find $x_i$ and replace $R$ by this number.

Step 4: If $R$ is unsatisfactory as a root estimate, repeat step 3. (Or repeat step 1 if $|R| \ll 1$.)
Citations


[p. 55. Lines 4–6.]

In fact, this result is part of a large subject called quadra-ture problems that interested readers can find more about in

...

[p. 744. Lines 18–20.]

Our general setting is Euclidean $n$-space $\mathbb{R}^n$ (with $n > 2$), equipped with the standard inner product $\langle \cdot, \cdot \rangle$ and the induced norm $\| \cdot \|$. 

81
[p. 8. Lines 2–4.]
Therefore, a normed linear space is really a pair \((E, \| \cdot \|)\) where \(E\) is a linear vector space and \(\| \cdot \| : E \to (0, \infty)\) is a norm. In speaking of normed spaces, we will frequently abuse this notation and write \(E\) instead of the pair \((E, \| \cdot \|)\).

Since the series in (3.2) generally has an infinite number of terms and the discrete parameter $m$ of (1.1) or (1.2) has been replaced by a continuous $W > 0$, a different notation is used in the following:...
Until now we have discussed the shape of a function on its whole domain, even though it is useful and perhaps more natural to work locally. For example, it is not difficult to show that $y = x^3 - 3x$ has folds at the points $x = \pm 1$ (i.e., is equivalent to $y = x^2$ in a neighborhood of each point) and in a neighborhood of every other point is a homeomorphism. On the other hand, this function is globally equivalent to neither a homeomorphism nor a (single) fold.
Citations


[p. 373. Theorem 2.]

**Theorem 2.** For any real numbers $x$ and $y$, $P[x]P[y] = P[x + y]$. 

[p. 375. Theorem 5.]

**Theorem 5.** For every real number \( x \), \( P[x] = e^{xL} \).

[p. 142. Lines 6–9.]

We shall entirely pass over several of Ramanujan’s contributions to the theory of numbers, including his investigation of the expression of integers in the form $ax^2 + by^2 + cz^2 + dt^2$ and his theory of highly composite numbers...

[p. 316. Lines 2–3.]

Unless specified otherwise, $\kappa$ denotes an uncountable regular cardinal and $\lambda$ is a cardinal $\geq \kappa$. 

88

[p. 285. Lines 12–10 from bottom.]

In this paper we therefore focus on the absolute contractivity properties of linearly implicit methods (i.e., $b_k^{(i)} = 0$ for $i = 1, \ldots, s - 1$) with fixed and variable step length.
Citations


[p. 744. Lines 2–6.]

A function \( f : X \to Y \) is closure continuous at \( x \in X \) provided that if \( V \) is an open set in \( Y \) containing \( f(x) \), there exists an open set \( U \) in \( X \) containing \( x \) such that \( f(\bar{U}) \subseteq \bar{V} \).

(Here context makes in which topological spaces the closure operations are taken; i.e. the bar on \( U \) refers to closure in \( X \) while the bar on \( V \) refers to closure in \( Y \).)
Citations


[p. 444. Lines 12–14.]

Definition. A tower is a finite sequence $A = (α_1, α_2, \ldots, α_n)$ of nonzero complex numbers such that for all $i \in \{1, 2, \ldots, n\}$ there exists some integer $m_i > 0$ such that $α_i^{m_i} \in A_{i-1}$ or $e^{α_i m_i} \in A_{i-1}$ (or both).
Citations


The debate over priority for Newton’s method may now be settled, but almost forgotten in the discussion is that Newton presented his method for approximating real roots side by side with a similar method for writing $y$ in terms of $x$ when $y$ is implicitly defined in terms of $x$ by a polynomial equation — a so-called “affected equation”.

92
A variable is free in a given expression (in which it occurs) if the meaning or value of the expression depends upon determination of a value of the variable; in other words, if the expression can be considered as representing a function with that variable as argument. In the contrary case, the variable is called a bound (or apparent, or dummy) variable.

[p. 106. Lines 4–1 from bottom.]

The binary operation in $S$, which we now denote by $\circ$, is completely determined by that in $T$, which we denote by juxtaposition, and the mappings $\psi$ and $\theta$ as follows (wherein $x, y \in T^0$, $t \in T$; $\kappa, \lambda \in K$):
In introductory courses in differential equations a certain process, the so-called method of undetermined coefficients, is often employed to find a particular integral of the linear differential equation with constant coefficients

\[ f(D)y = (D^n + a_1 D^{n-1} + \cdots + a_{n-1} D + a_n) y = r(x). \]

[p. 480. Lemma 1.]

Suppose that the cubic graph $G$ arises as just described from a complete game of Sprouts played on $m$ vertices in $p$ plays. Then

$$f = 2 + p - m$$


We will use the power notation to denote repeated composition. Formally, $f^0 = I$ and for $n \geq 0$, $f^{n+1} = f \circ f^n$. 
  [p. 408. Lines 14–20.]

We would like to say that one algorithm is faster, uses less time, than another algorithm if when we run the two algorithms on a computer the faster one will finish first. Unfortunately, to make this a fair test, we would have to keep a number of conditions constant. For example, we would have to code the two algorithms in the same programming language ...

98
We establish some notation and conventions that will be used throughout the paper.

[p. 524. Lines 17–16 from bottom.]
In the following exercises students have to use ruler and pencil to work on graphs and curves given by drawings.
A word is a finite sequence of elements of some finite set $\Sigma$. We call the set $\Sigma$ the alphabet, the elements of $\Sigma$ letters. The set of all words over $\Sigma$ is written $\Sigma^*$.

... The empty word, with no letters, is denoted by $\varepsilon$. 


[p. 539. 5.2.7.]
On \{r_4 \leq r \leq r_5\} \text{ and } z \leq z_2\} the action of f on z-coordinates is multiplication by \(e(r)\), where \(e(r)\) is a smooth bump function with graph shown in Figure 4.

[p. 729. Line 23.]

Then $\phi(0) = m_1 (\lfloor U \cap F \rfloor \cap \lfloor U \cap G \rfloor) > .8\epsilon$. 

103
104. (37, 133, 171, 175, 185, 211, 261, 280) **Davey, B. A. and H. A. Priestley (1990), Introduction to Lattices and Order.** Cambridge University Press.

[p. 3. Beginning of 1.4.]

Each of \( \mathbb{N} \) (the natural numbers \( \{1, 2, 3, \ldots \} \)), \( \mathbb{Z} \) (the integers) and \( \mathbb{Q} \) (the rational numbers) also has a natural order making it a chain.

[p. 139. Lines 12–14.]

R. K. Dennis has shown that $-d\log \{u, v\} = (uv)^{-1} du \wedge dv$ implies that $-d\log (\langle a, b \rangle) = (1 + ab)^{-1} da \wedge db$ for all pointy brackets $\langle a, b \rangle$. 
Citations


[p. 795. Line 6 from bottom.]

In this situation, it is convenient to introduce the new variables

\[(u, v) := T x = (\xi^T x, \eta^T x).\]
Euler's constant $\gamma$ is usually defined by the limit relation
\[ \gamma = \lim_{n \to \infty} D_n, \]
where
\[ D_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log n. \]
Citations


[p. 222. Lines 10–9 from bottom.]

That is, if a point is contained in $J(F)$, then so are all of its images and all of its preimages.

[p. 233. Lines 1 and 8.]
Let first $k = 0$, i.e, $m = n$. . . . Now let $k > 0$. . . .
A monoid \([M, \circ]\) with identity element 1 is a group iff for each \(m \in M\) there is an inverse element \(m^{-1} \in M\) such that\
\[
m^{-1} \circ m = m \circ m^{-1} = 1
\]

[p. 546. Lemma 1.1.]

If $C$ is r.e. in $X$ but not recursive in $X$, then there is an ordering $A$ such that $A$ is recursive in $C$ and no copy of $A$ is recursive in $C$. 

[p. 258. First line of Theorem.]

Let $D$ be a domain, $a, b \neq 0$ two complex numbers and $k \geq 1$ an integer.
Citations


[p. 353. Line 6.]

Proof. For each \( d \), \( u(d, \cdot) \) has an inverse function \( u^{-1}(d, \cdot) \), where

\[ u(d, c) = t \leftrightarrow u^{-1}(d, t) = c. \]
...it can be seen that the number of representations of $n$ as a sum of four squares is eight times the sum of those divisors of $n$ which are not multiples of four. In particular, it is never zero!
Citations

   [p. 1419. Last line of Definition 1..]
   We denote the circle diagram by enclosing the string diagram in square brackets, e.g. [XOXOX].

[p. 413. Lines 13–14.]

However, the singular zeroes of a harmonic function are not always isolated.
Citations


(The assumption that such discrete data can be plotted as a continuous curve is often referred to as the continuum hypothesis.)

[p. 7. Lines 6–5 from the bottom.]

If $v(t)$ is the moment curve, then we may calculate $\|\gamma'(t)\|$ with the help of the following observations and some messy algebra:

[p. 574. Problem 10426.]

Show that any integer can be expressed as a sum of two squares and a cube. Note that the integer being represented and the cube are both allowed to be negative.

[p. 257. Lines 14–16.]

Setting

\[ f(t) = c_{q-1}t^{q-1} + \cdots + c_1t + c_0, \]

we then have \( f(\xi) = 0 \). Furthermore, \( f(t) \) has integral coefficients.
The theory of gravitation is determined by the following formal elements:

1. The four-dimensional space-time continuum.
2. The covariance of the field equations with respect to all continuous coordinate transformations.
3. The existence of a Riemannian metric (i.e. a symmetric tensor $g_{ik}$ of the second rank) which defines the structure of the physical continuum.

Under these circumstances the gravitational equations are the simplest conditions for the functions $g_{ik}$ which restrict them to a sufficient degree.

We have tried to retain 1) and 2) but to describe the space structure by a mathematical object different from 3) yet resembling in some way the $g_{ik}$. 


[p. 1. Lines 8–17.]

[p. 786. Lines 7–6 from bottom.]

7.5 Lemma. For any uncountable subset $P'$ of $P$ there is a free subgroup $A'$ which is pure in $A$ and an uncountable subset $P''$ of $P'$ such that $\text{dom}(\phi) \subseteq A'$ for every $\phi$ in $P'$.
Citations


[p. 147. Lines 8–11.]

If $\alpha = \beta = 0$, we have the Legendre-polynomials $P_n(x)$; for $\alpha = \beta = \frac{1}{2}$ the Tschebischeff polynomials $T_n(x)$. 

[p. 35. Theorem 1.115.]

Suppose $E$ is an equivalence relation on a set $S$. For any $x$ in $S$, denote by $E_x$ the set of all $y$ in $S$ equivalent under $E$ to $x$. Then the collection of all $E_x$ is a partition of $S$. 
The following inequalities hold for any vector $\eta$ tangent to a leaf of the foliation $F$ of $FM$.

$$2h(dq(\eta), dq(\eta)) \geq \hat{h}(\eta, \eta) \geq h(dq(\eta), dq(\eta))$$

where $q : DM \to M$ denotes the bundle projections.
Given a set of symbols $S = \{x_1, x_2, \ldots, x_n\}$, the set

$$G = \{x_1, x_2, \ldots, x_n, (x_1x_1), (x_1x_2), \ldots, (x_nx_n), x(x_1(x_1x_1)), (x_1(x_1x_2)), \ldots, (x_1(x_nx_n)), ((x_1x_1)x_1), ((x_1x_1)x_2), \ldots \}$$

obtained by repeated juxtaposition of the symbols already written down, forms a groupoid in a natural way (the binary operation being juxtaposition).

[p. 381. Second to last displayed formula.]

\[
1 \leq \frac{L_n^2}{d_n^2} \leq \frac{[1 + c(a_{n+1} + a_n)]^2}{1 + c^2(a_{n+1} + a_n)^2} \rightarrow \frac{c^2}{c^2} = 1
\]
Citations


[p. 774. Formula (11) and the line above.]

The same procedure with $\Omega^* = \Omega$ yields

$$2H = \frac{|\Sigma| \cos \gamma}{|\Omega|}$$


(i) if $f$ is in the domain of $T$, and $\text{dom}(f)$ is the domain of $f$, then the function with domain $\text{dom}(f)$ and constant value 1 is in the domain of $T$ . . .
We define the identity function $I$ by letting $I(x) = x$ for each real number $x$. 


[p. 394. Line 15.]

We define the identity function $I$ by letting $I(x) = x$ for each real number $x$. 

134
We shall actually solve the following equivalent problem: In the set \( \{ w_i > 0; \Sigma w_i = 1 \} \), find values \( w_i \) that maximize

\[
F_A(w) = \prod_{i<j} \left( \frac{w_i}{w_i + w_j} \right)^{a_{ij}} \left( \frac{w_j}{w_i + w_j} \right)^{a_{ji}}
\]

[p. 79. Lines 11–9 from bottom.]

In a data fitting code [8] written at the Lockheed Aircraft Company, for example, $p_i(x)$ was selected to be the $i$th Chebyshov polynomial $T_i(x)$ over an interval containing all the $x_\mu$. 

[p. 377. Line 9 above picture.]

To begin, recall that an isometry of $\mathbb{R}^3$ is a bijection which preserves distance.
The factorial function can be defined by $x! = \Gamma(x + 1) = \int_{0}^{\infty} t^x e^{-t} dt$ for $x > -1$, extended uniquely to negative non-integral $x$ by defining $x! = (x+n)!/(x+n)(x+n-1)\ldots(x+1)$ for any integer $n$ such that $(x+n) > -1$. 


[p. 2. Lines 1–4 of Section 2.]
Finally, we remark that for $\phi : A \rightarrow B$, the set $A$ is the domain of $\phi$, the set $B$ is the codomain of $\phi$, and the set $A\phi = \{a\phi \mid a \in A\}$ is the image of $A$ under $\phi$. 
Richard Darst and Gerald Taylor investigated the differentiability of functions $f^p$ (which for our purposes we will restrict to (0, 1)) defined for each $p \geq 1$ by

$$f(x) = \begin{cases} 
0 & \text{if } x \text{ is irrational} \\
1/n^p & \text{if } x = m/n \text{ with } (m, n) = 1.
\end{cases}$$

[p. 614. Formula (4).]

... if $p > 1$ and $B$ is the set of numbers $x$ that are not dyadic rationals and satisfy

$$|x - m/2^n| \leq (2^n)^{-p}$$

for infinitely many dyadic rationals $m/2^n$, then...

[p. 705. Abstract.]

We call a set right scattered (left scattered) if every non-empty subset contains a point isolated on the right (left).
Theorem 3. If $F$ is an interval function satisfying condition (A) on an open interval $I$ such that …

Proof. Let $F$ be as stated and let $A$, $B$ and $C$ be disjoint sets where $A \cup B \cup C = I$…

[p. 33. Lines 13–10 from bottom.]

In general, let \((X_i, \mathcal{B}_i, m_i)\) be measure spaces with \(m_i(X_i) = 1, i = 1, 2\). Let \(\phi\) be an invertible mapping from \(X_1\) to \(X_2\) such that \(m_1(B_1) = m_2(\phi(B_1)), B_1 \in \mathcal{B}_1, \) and \(m_2(B_2) = m_1(\phi^{-1}(B_2)), B_2 \in \mathcal{B}_2.\) We refer to \(\phi\) as an isomorphism. Transformations \(T_i\) on \(X_i, i = 1, 2\) are isomorphic if there exists an isomorphism \(\phi\) such that \(T_1(x) = \phi^{-1}(T_2(\phi(x)))\) for \(x \in X_1.\) We refer to \(T_2\) as a copy of \(T_1.\)
Citations


[p. 407. Lines 10–7 from bottom.]

We may, however, add an extra character to the alphabet, giving \((\lambda + 1)\) characters, construct a table without transpositions, and then eliminate all words containing the extra character.
Recall that a relatively minimal simply connected elliptic surface $S$ is specified up to deformation type by its geometric genus $p_g(S)$ and by two relatively prime integers $m_1, m_2$, the multiplicities of its multiple fibers.

[p. 842. Lines 17–20.]
By applying the Mean Value Theorem to $f$ on $[a, d]$ and $[d, b]$ respectively, we obtain

$$f(d) - f(a) = f'(c_1)(d - a), \quad c_1 \in (a, d),$$

$$f(b) - f(d) = f'(c_2)(b - d), \quad c_1 \in (d, b).$$

[p. 247. Lines 1–3.]
Material implication, the logical connective corresponding to the English ”if” . . .
It is the thesis of this paper that this uneasiness is none other than the familiar temptation to commit the fallacy of conversion \((p \Rightarrow q \mid \neg q \Rightarrow p)\), also known as the fallacy of affirming the consequence \((p \Rightarrow q, q \mid \neg p)\) . . .

[p. 352. Lines 1–2.]

Let $P$ be a finite partially ordered set: a *chain* (*antichain*) in $P$ is a set of pairwise comparable (incomparable) elements; the *width* of $P$ is the maximum cardinality of an antichain in $P$. 

[p. 844. Title.]
Block Structure in the Chebyshev-Padé Table

Show that for $f$ in Exercise 2.1.1.13. 49, if $g$ is given by

$$g(x) \overset{\text{def}}{=} \int_0^x f(t)dt$$

then: ...
Citations


[p. 85. Theorem 2.1.3.5..]

Let (2.1.3.3) obtain everywhere on a measurable set $E$ of positive measure. Then ...
In other words, \( y = 0 \) is always a boundary point of \( Y \). This turns out to be true in general when \( (P) \) has a finite optimal value but is unstable, as follows from the next theorem.
Suppose we are given a smooth simple closed plane curve and a point \( p \) on it. Is there always a circle or straight line tangent to the curve at \( p \) and at another point \( p' \neq p \)?
2 Definition. A function $g$ is of type (A) on $I$ if $g$ has an antiderivative $v$ on $I$ and

$$g(a) = g(b) \text{ implies } v(a) = v(b) \text{ for all } a \text{ and } b \text{ in } I$$
Citations


[p. 622. Lines 5–3 from bottom.]

The nu relation between symbols is such that the following sentences are true:

\[ 2 \nu \text{ odd}, \quad 4 \nu \text{ odd}, \quad 1 \nu \text{ even}, \quad 3 \nu \text{ even}. \]

[p. 693. Line 1.]

It was the plan of Tschebyshef [1] and Sylvester [2] to deduce the prime number theorem . . .

[p. 571. Lines 6–7 under (9).]

Projection translations are first order translations whose translation formula is *projective*, i.e., of a special, particularly simple form.

[p. 44. Lines 12–13.]

Basically, what Higgins found when $a = 0$ in his general formula (2.5) is

$$B_n = \sum_{k=0}^{n} \frac{1}{k+1} \sum_{j=0}^{k} (-1)^j \binom{k}{j} j^n, \quad n \geq 0,$$
Let $h = \{ x + iy \mid y > 0 \}$ be the complex upper half-plane.
Theorem. Let $G$ be a graph, let $\epsilon > 0$ and let $k$ be a positive integer. Then there exists a constant $K$, depending on $k$ and $\epsilon$ only, such that the vertices of $G$ can be partitioned into $m$ sets $A_1, \ldots, A_m$, with $k \leq m \leq K$, such that at least $(1 - \epsilon) \left( \frac{m}{2} \right)$ of the pairs $(A_i, A_j)$ (with $i < j$) are $\epsilon$-uniform.

[p. 665. Lines 4–3 from bottom.]

Assume \( \phi \) has no fixed vertex. Then, for every vertex \( v \), there is a unique non-trivial path in \( T \) from \( v \) to \( \phi(v) \).
Let $f(x)$ be a continuous, monotonically increasing function with the property that 

$$f(x) = \text{integer} \Rightarrow x = \text{integer}$$

(The symbol ‘$\Rightarrow$’ means “implies.”)

[p. 589. Line 5 from bottom.] When $X = X_c$, $\mu \ast f$ is well-defined for every $\mu \in M(G)$ and every $f \in X$. 

165
This is the usual picture of the Lie bracket $[X, Y] = Z$, and becomes especially clear if drawn in a flowbox for $X$. 


[p. 304. Lines 7–6 from bottom.]
A is a proper subset of $B$ if $A$ is a subset of $B$ but $A$ is not equal to $B$. If $A$ is a proper subset of $B$, we write $A \subset B$. 

Citations


[p. 234. Definition 5.3.]

[p. 128. Example 3.2(a).]

\[ \text{A} = \{1, 4, 9, \ldots, 64, 81\} = \{x^2 \mid x \in \mathcal{U}, x^2 < 100\} = \{x^2 \mid x \in \mathcal{U} \land x^2 < 100\} = \{x^2 \in \mathcal{U} \mid x^2 < 100\}. \]

[p. 36. Footnote.]

All logarithms in this text are base two unless noted otherwise. We use the now quite common notation $\lg x$ instead of $\log_2 x$. 

[p. 692. Lines 15–18.]

... because the construction of \( \text{conv } E \) involves global properties of \( E \), appropriate smoothness of the boundary of \( \text{dom}(E) \), the domain of \( E \), may have to be required near the points of some subset of \( \partial (\text{dom}(E)) \cap \text{dom}(E) \).

Notice that the proof format makes it easy to find the facts on which the proof depends – they are given within the angle brackets ⟨ and ⟩.
Consider a surface $S$ and a point $x$ on $S$. Let the parametric vector equation of $S$ be

$$x = x(u,v).$$

The ambient space of the osculating planes at the point $x$ to all of the curves through $x$ is a certain space $S(2,0)$ called the two-osculating space of $S$ at $x$. 


[p. 627. Lines 1–5.]

[p. 537. Lines 16–17.]

1. A connected compact *abelian* Lie group is necessarily a *torus*, by which is meant a direct product of circles.

[p. 210. Lines 5–8.] Because $K$ is a function, a unique output $v$ exists for each input $u$ in the domain of $K$ and if $K$ is in some sense *continuous*, as is very often the case, the output $v$ depends continuously on $u$. 
Our third result examines maps that have “sensitive dependence to initial conditions”. These are maps whose non-wandering set contains an interval.

[p. 308. Lines 1–3.]

Let $Y = [0, 1]$ with the usual topology, and let $Z$ be a non-Borel subset of $Y$, which by the axiom of choice exists. Let the elements of $Z$ be denoted by $\{y_\lambda : \lambda \in \Lambda\}$, $\Lambda$ an index set.
We may think of them as

\[ x_k = \bar{\phi}_k(x_1, x_2) = \phi_k \left( (x_1 + ix_2)^{\frac{1}{m}} \right), \quad k = 3, \ldots, n, \]

where \( \bar{\phi}_k \) is a multi-valued function of \( x_1, x_2 \), defining an \( m \)-sheeted surface over a neighborhood of the origin.

[p. 328. Lines 11–6 from bottom.]

In this section we assume that the field $F$ is either the real or complex numbers. . . . Throughout this section let $\overline{E} = \{e \in E : \|e\| = 1\}$, and for any subset $B$ of $Y$, let $\overline{\sigma}(B)$ denote the $\sigma(Y, X)$-closed convex hull of $B$ in $Y$. 

178
We need to distinguish a Boolean algebra from the general concept of, or the formal theory of, Boolean algebras. When using the indefinite article, we are referring to a particular mathematical structure which, via an appropriate interpretation, satisfies the above axioms.

[p. 32. Lines 17–18.]

Let $\psi'_h$ be $\psi_h$ with its codomain extended from $S_h$ to $\text{Sym}M$ . . .
Citations


[p. 365. Last two lines.]
Suppose, moreover, that \( A \) and \( N \) are two distinct objects not contained in \( S \); intuitively \( A \) and \( N \) are to be thought of as the connectives “and” and “not”.

181
The simplest statement of Cantor’s continuum hypothesis...
183. (9, 101) **Halbeisen, L. and S. Shelah (2001),**

[p. 239. Lines 15–16.]

We will use fraktur-letters to denote cardinals and ℵ’ s to denote the alephs.

[p. 639. Lines 9–8 from bottom.]

Following the pattern first observed by Waldspurger, the vanishing of $\Theta(\pi, \omega)$ to $GU_{HE}(D)$ either vanishes or equals $(\pi^D, \omega^{-1}), \ldots$

[p. 559. Lines 1–2.]

A Gaussian integer, $\gamma$, is a complex number that can be expressed as $\gamma = a + bi$, where $a$ and $b$ are real integers and $i$ is the so-called imaginary unit.
Then we can identify the group \( B_\sigma \) with the group \( P(\sigma) \) consisting of the set \( \sigma! = \{ f^* \mid f \in B_\sigma \} \) under the multiplication “\( \cdot \)” defined by \( f^* \cdot g^* = (fg)^* \).
Applying Newton’s law of motion, we obtain the ordinary differential equation
\[ ml\ddot{u} + c\dot{u} + mg\sin u = 0, \]
where \( c \) is the positive constant of proportionality.

[p. 564. Lines 5–2.]

[p. 772. Lines 4–6.]

To find the critical points of a smooth function \( f \) defined on \( M^n \subset \mathbb{R}^{n+m} \), a smooth submanifold given as the common zero-set of \( m \) smooth functions \( g_i : \mathbb{R}^{n+m} \rightarrow \mathbb{R} \).

[p. 774. Lines 17–20.]

Fact 2. Let \( b \) be a non-degenerate symmetric bilinear form on \( V \) of dimension \( 2m \) and let \( W \subset V \) have dimension \( m \) and \( W \subset W^\perp \). Then \( b \) is represented by the matrix
\[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

Proof: Choose \( \vec{w} \neq 0 \) in \( W \) and \( \vec{v} \) such that \( b(\vec{v}, \vec{w}) = 1 \).

...
In each case, the extension defined on $R^*$ must be identical on $R$ with the original function or relation defined on the field $R$, and of the same arity.

[p. 162. Lines 4–6.]
The labors of Gauss, Kummer, Dirichlet, Kronecker, Dedekind, and others, have extended the scope of the theory of numbers far beyond its original limit of the science of the natural numbers 0, ±1, ±2, ±3, . . . .

[p. 333. Lines 10–9 from bottom.]

... to obtain the 2nd order differential equation

\[
\left(1 + (y')^2\right) (1 + \mu y') + 2 (y - \mu x) y'' = 0.
\]

[p. 2. Lines 9–8.]

Suppose $R$ is a commutative ring with identity and the $R$-module $M$ is the direct sum of a family $\{Rx_k : k \in K\}$ of cyclic submodules.

[p. 47. Lines 3–4.]

**Definitions 1.** (i) A *type* $\tau$ is a set of operation symbols, each with an assigned *arity* (some cardinal). $\tau$ is *bounded* by the infinite regular cardinal $\alpha$ if the arities of its operation symbols are all $< \alpha$. 
Citations


[p. 213. Lines 6–10 under Proposition 5.1.]

... QF(X) is an F-space if and only if:

(*) If C1, C2 are disjoint cozero sets, then there are zero sets Z1, Z2 such that C1 ⊆ Z1, C2 ⊆ Z2 and int (Z1 ∩ Z2) = ∅.

Thus (*) is a sufficient but not necessary condition for QF(X) to be an SV-space.
The set $A$ will be said to be a *subset* of the set $S$ if every element in $A$ is an element of $S$, that is, if $a \in A$ implies that $a \in S$. We shall write this as $A \subset S$ . . . This notation is not meant to preclude the possibility that $A = S$. 


[p. 2. Lines 11–14.]

[p. 440. Lines 10–7 from bottom.]


[p. 213. Lines 16–12 from bottom.]

Addition of products

<table>
<thead>
<tr>
<th>English statements</th>
<th>Arithmetic statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two times three plus three times four</td>
<td>$2 \times 3 + 3 \times 4$</td>
</tr>
<tr>
<td>Two times four plus three times six</td>
<td>$2 \times 4 + 3 \times 6$</td>
</tr>
</tbody>
</table>

[p. 241. Lines 12–9 from bottom.]

It follows that $y'(x)$ is never increasing for $a < x$ and $y(x)$ is concave downwards. Since the graph of $y = y(x)$ lies below the curve tangent and does not intersect the $x$-axis for $x > a$, we must have $y'(x) > 0$. 
We shall refer to a continuous transformation as a *mapping* from now on.

By repeated application of this squaring method, polynomials \( k P_n(z) = \sum_{j=0}^{n} k a_{n-j} z^j \) can be obtained, the roots of which are equal to \( -x_i^k \), and the coefficients are determined recursively by

\[
k a_j = k-1 a_j^2 + 2 \sum_{i=1}^{j} (-1)^i k-j+1 a_{j+1} k-1 a_{j-1}
\]

[p. 685. Lines 9–4.]

This paper has the following organization. §1 contains the definitions and notations that will be used. §2 contains some technical results on Poincaré complexes and the central lemma which makes everything work and which we call the Surgery Preparation Lemma. The remaining paragraphs unwind the details, so that §3 contains the proof of the ($\pi - \pi$) theorem for Poincaré duality spaces of formal dimension $\geq 7 \ldots$
Citations


[p. 630. Lines 19–18 from bottom.]

... and thus $G_0/K_0$ is homeomorphic to a euclidean space only if $C = K_0$. 

203

[p. 206. Lines 5–4 from the bottom.]
Thus \((M + M^\perp)^\perp = 0^\perp = E\). But \(M + M^\perp\) is closed, so \(M + M^\perp = E\).
R. Hunt proposed the problem of finding an analogue of Dahlberg’s result for the heat equation in domains whose boundaries are given locally as graphs of functions $A(x, t)$ which are Lipschitz in the space variable. It was conjectured at one time that $A$ should be $\text{Lip}_1$ in the time variable, but subsequent counterexamples of Kaufmann and Wu [KW] showed that this condition does not suffice.
Citations


[p. 222. line 10 from bottom.]

The set of those elements of $L$ which are the join of finitely many atoms is closed under the operations $\lor$ and $\land$ . . .

[p. 8. Lines 7–9.]

Thus our computation of the action of $p$ on individual $K$-types is in principle (and will turn out to be in practice) sufficient for understanding the submodule structure of $S^a(X^0)$. 

[p. 2385. First line of proof of Lemma 10.2.]
We will prove the contrapositive statement.

[p. 56. Lines 9–6 from bottom.]

But in modern algebra texts published since 1986 (and a few earlier ones), the reader should be more careful. Even when the Lemma is stated, its proof may be faulty. Most of these books present either a fallacious proof of the Theorem or Lemma (counterexample above) or a suspiciously incomplete proof.

[p. 3934. Lines 8–6 from bottom.]

We will often abuse notation by omitting mention of the natural isomorphisms making $\wedge$ associative and unital.

[p. 905. First lines.]

Definition 2.1. For nonzero subspaces $\mathcal{R}, \mathcal{N} \subseteq \mathbb{R}^n$, the minimal angle between $\mathcal{R}$ and $\mathcal{N}$ is defined to be the number $0 \leq \theta \leq \pi/2$ that satisfies

$$\cos \theta = \max_{u \in \mathcal{R}, v \in \mathcal{N}} \frac{v^T u}{\|u\|_2 \|v\|_2}$$

[p. 70. Lines 13–12 from bottom.]

It must be recognized however that the series has not yet been proved to converge to the value \( f(x) \).

[p. 55. Definition of permutation.]

Let $X$ be a set with $n$ different objects. An arrangement of all the elements of $X$ in a sequence of length $n$ is called a permutation.

A small library contains 15 different books. If five different students simultaneously check out one book each, . . .
An integral equation of the form
\[ \int_0^a K_0(t, \rho)g(t)\,dt = f(\rho), \quad 0 < \rho < a, \]
with a given symmetric kernel \( K_0 \), embodies the solution of various mixed boundary value problems of physical interest. Being a Fredholm integral equation of the first kind, it cannot, in general, be solved easily for the unknown function \( g(\rho) \) in terms of the given function \( f(\rho) \).

[p. 237. Lines 10–8 from bottom.]

A continuous function $F$ with real number values and argument in a Banach space $B$ is a *real linear functional*, or simply *linear functional*, if $f(x + y) = f(x) + f(y)$ for each $x$ and $y$ of $B$. 

[p. 300. lines 17–14 from bottom.]

By taking \( I = J \) in the above theorem, the following corollary answers the question about the relationship between \( \tau^* \) and \( \tau^{**} \).

[p. 636. Lines 4–8.]

A mathematical neural network model is an $n$-dimensional difference equation or differential equation dynamical system that accounts for the dynamics of $n$ neurons. Each neuron is mathematically a state $x_i$ (a real number) with an associated output $g_i = g_i(x_i)$, typically a ramp or sigmoidal gain function $g_i(x_i)$. 

[p. 634. Line 1.]

A triangular triple is a sequence of non-negative integers \((i, j, k)\) that gives the lengths of the sides of a triangle.

[p. 10. Lines 4–6.]

This computation is to be done under the assumption that individuals’ birthdays are independent and that for every individual, all 365 days of the year are equally likely as possible birthdays.

[p. 415. Lines 7–2.]

In order to avoid a prohibitive amount of punctuation, we assign the following order of precedence to the binary symbols $\equiv, \land, \cdot$. Of these, each has precedence over any listed to its right. To the unary symbols $E_1$ and $E_2$ we assign the least precedence. When two or more unary symbols occur in a sequence, the innermost symbol is given the least precedence.

[ p. 95. Lines 16–17. ]

Definition. A positive integer \( n > 1 \) is perfectly symmetric if its reciprocal is symmetric in any base \( b \) provided \( b \not\equiv 0 \) (mod \( n \)) and \( b \not\equiv 1 \) (mod \( n \)).

[p. 765. Lines 8–10 below Figure 15.]

Let $i, j, k$ be the rotations of $S_1$ around the coordinate axes $x, y, z$, respectively, through $\pi$ radians. Let us define a group generated by $i, j, k$ modulo the equivalence relation of an ambient isotopy in a hollow ball $H$ with the band $A$ keeping $S_1$ and $S_2$ fixed.

Under these conditions adding the relations in (2.6) using obvious inequalities produces

\[(2.8) \quad x' + y' < 2 \frac{xy + \frac{1}{2}(x + y)}{1 + \frac{1}{2}(x + y)}\]

Since \(4xy \leq (x + y)^2\) we see that \(x' + y' < x + y\). It follows that \(x^{(n)} + y^{(n)}\) decreases in \(n\) and its limit is necessarily zero indicating that 0 is globally stable.
Citations


[p. 479. Lines 11–13 of Introduction.]

The main idea here is to characterize those twisted sums of Köthe function spaces which can be obtained by differentiating a complex interpolation scale of Köthe function spaces.
From the definition of $T$, we may interpret the elements in $T$ as those invertible elements in $\mathbb{Z}/e'\mathbb{Z}$ that are of the form $1 + kd$ for some $k$ because ...
What about special values? By construction, we have, for $k \geq 1$,
\[ L(E, \lambda \omega) (\chi_k) = 2J_k (E, \lambda \omega) = 2G_k (E, \lambda \omega) - 2p^{k-1} (\text{Frob}G_k) (E, \lambda \omega), \]
a formula we will be able to unwind only in special cases.

[p. 429. Lines 8–10.] Because sets of small Lebesgue measure have small $\mu_k$-measure, a proper choice of $Y = \eta_{k+1}$ enables us to obtain the necessary estimates.
   [p. 273. Lines 13–11 from bottom.]
   Usually, a function $f$ on $\mathbb{R}^2$ or $\mathbb{R}^3$ is determined from its integrals over lines or planes. Here $f$ represents some internal property of an object which cannot be observed directly, while some integrals of $f$ are observable.
Citations


[p. 145. Lines 1–2.]

In calculus, we encounter the result that the composition $g \circ f$ of two continuous functions $f : I \to I$ and $g : I \to I$ is continuous, where $I = [0, 1]$. 

230

[p. 166. Fourth paragraph.]
Parametrize $y = x^2$ by $x = t$, $y = t^2$, and plug into $Ax + By$. 

231

[p. 63. Lines 11–9.]

Dilworth’s Theorem [D] asserts than any partial ordering of finite width \( n \) can be covered with \( n \) chains. If the partial ordering is finite, then one can actually exhibit these chains (by trial and error, if by no other method).

[p. 668. Line 1.]
Let $f$ be a strictly increasing function with continuous derivative on a compact interval $[a, b]$. 
The largest region leaves in one iteration and is bounded by the curves

\[ x_{1_i} = \frac{1}{2} + (-1)^i \sqrt{\frac{1}{4} - \frac{1}{a}}, \quad i = 1, 2 \]

which satisfy \( f_a(x) = 1 \).

[p. 417. First two lines of paragraph (b).]

The polynomial rings $\mathbb{R}[x]$ and $\mathbb{R}[x, y]$ in one and two variables, respectively, share important properties but also differ in significant ways.

[p. 678. Lines 18–20.]
This says (in our terminology) that if $E$ is the splitting field of a polynomial $f(x)$ over a field $F$, then $E = F(V)$ for some rational function $V$ of the roots of $f(x)$. 

[p. 757. Problem A-2.]

Define $C(\alpha)$ to be the coefficient of $x^{1992}$ in the power series expansion about $x = 0$ of $(1 + x)^\alpha$. Evaluate ...

[p. 758. Problem B-1.]

Let $S$ be a set of $n$ distinct real numbers.
When is $x^y$ less than $y^x$? For what kind of numbers does $x^y = y^x$? And is there a formula for $y$ as a function of $x$?

[p. 732. Lines 2–7.]

For example, the quadratic form $x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_2x_3$ can be specified either by the triangular matrix

$$
\begin{pmatrix}
1 & -2 & -2 \\
0 & 1 & -2 \\
0 & 0 & 1
\end{pmatrix}
$$

or by the symmetric matrix

$$
\begin{pmatrix}
1 & -1 & -1 \\
-1 & 1 & -1 \\
-1 & -1 & -1
\end{pmatrix}
$$
Let $R$ and $S$ be relations from $A$ to $B$. If $R(a) = S(a)$ for all $a$ in $A$, then $R = S$. 


[p. 111. Theorem 2.]
Define the binomial coefficient of the first kind \( \binom{n}{k} \) to be the number of \( k \)-element subsets of \( S \); that is, the number of ways to choose \( k \) distinct objects from \( S \) with the order of selection not important.

[p. 904. Lines 1-3.]

For $x \in X$ let $N(x)$ denote the intersection of all open neighborhoods of $x$, which of course is not a neighborhood in general.

[p. 93. Lines 9–11.]

If for each $i \in I$, $A_i$ is a value semigroup (together with $+i$, 0, $\infty_i$), then so is their product (with $+$, 0 defined coordinatewise; 1/2 and inf are also taken coordinatewise.)
Dirichlet’s Principle. There exists a function $u$ that minimizes the functional (the so-called Dirichlet Integral)

$$D[u] = \int_{\Omega} |\text{grad } u|^2 \, dV, \quad \Omega \subset \mathbb{R}^2 \text{ or } \mathbb{R}^3,$$

among all functions $u \in C^1(\Omega) \cap C^0(\overline{\Omega})$ which take on given values $f$ on the boundary $\partial \Omega$ of $\Omega$. . . .

   [p. 36. Lines 8–9.]

   We make use of a type concept and an operator concept as well as the overloading of (certain) function names.
Conversely, if an arbitrary function \( f : X \to \mathbb{R} \) satisfies (1.6) and if the (finite Radon) measure \( \mu \) is complete, then \( f \) is \( \mathcal{A} \)-measurable.

[p. 305. Lines 11–12.]

A finite projective plane of order $n$, with $n > 0$, is a collection of $n^2 + n + 1$ lines and $n^2 + n + 1$ points such that ...
Let $A$ be a ring, and let $U$ be the set of elements of $A$ which have both a right and a left inverse. Therefore $U$ satisfies all the axioms of a multiplicative group, and is called the group of units of $A$. It is sometimes denoted by $A^*$, and is also called the group of invertible elements of $A$. 


[p. 56. Lines 9–2 from bottom.]
Fix some $s \in (0, 1]$, and consider the Cantor set $C_s$ obtained by removing an open interval of length $(1 - s)/2$ from the middle of $I$, open intervals of length $(1 - s)/8$ from the center of the two resulting open intervals, open intervals of length $(1 - s)/32$ from the four resulting intervals, and so on. The Cantor set $C_s$ has one dimensional Lebesgue measure $s$. There is a standard mapping of $C_s$ onto $I$, the Cantor function $\Phi_s \ldots$ which can be embedded as the last stage of a one parameter family of maps $t \rightarrow \Phi_{s,t}$. 


[p. 55. Proposition 3.2.]

**Proposition 3.2.** (a) $VN(G)^*$ has a right identity if and only if $G$ is amenable. (b) $VN(G)^*$ has a left identity if and only if $G$ is compact.

[p. 243. Proposition 1.2.]

The velocity of a marble rolling without friction down a ramp is proportional to $\sqrt{|y|}$, if the marble starts at rest at a point where $y = 0$.

[p. 924. Lines 6–5 from bottom.]

The key ingredient in the preceding discussion is the fact that for finite sets $S$, every injective $f : S \to S$ is necessarily surjective.

[p. 428. Corollary 13.]

**Corollary 13.** No vector space $V$ over an infinite field $F$ is a finite union of proper subspaces.

[p. 643. Lines 1–2.]

**Definition.** Let $\Phi(z, y)$ be the minimal polynomial for $F(x)$ over $Q(y)$. We call $\Phi(z, y)$ the Galois resolvent of $\Pi$ corresponding to $F(x)$. 

[p. 228. Lines 3–5.]

My answer was 23124110, and the story might have ended there; but though I do not know why, I continued: 1413223110, 1423224110, 2413323110, 1433223110, 1433223110, and then the sequence is constant from this term on.

[p. 216. Lines 21–23.]

Other algorithms may even give a nontrivial factor of $p$, ...
Citations


[p. 353. Title.]
A Simple Proof of Zorn’s Lemma.
We call two sets \( A \) and \( B \) equinumerous if there is a bijection \( f : A \to B \). [This statement also occurs on page 21 of the First Edition.]
For a given positive integer $n$, and a regular $C^2$ plane convex curve $\Gamma$ that includes no straight line segment, we wish to approximate $\Gamma$ by an inscribed $n$-segment polygonal line that minimizes the area between curve and polygonal line.
Citations


[p. 913. Lines 19–18 from bottom.]
For ease of notation, we will denote $\log_2 x$ by $\lg x$. 

264

[p. 243. Lines 4–5.]

The trace of the curve $\gamma$ in the complex plane is denoted by $\{\gamma\}$. 

[p. 753. Item (2) under III.]

There are three cases:
(1) $E$ has finite cardinality.
(2) $E$ has denumerable cardinality.
(3) $E$ has the cardinality $c$ of the continuum.

[p. 56. Chapter 1, verse 48.]

... from henceforth all generations shall call me blessed.

[p. 547. Lines 9–11.]

Our objective in the remainder of this section is to develop the $\mathcal{U}$-homotopy necessary to replace a general, perhaps pathological, function $f$ by a more manageable and regular function $h$. 

[p. 58. Line 15.]

Thus a universal element \( \langle r, e \rangle \) for \( H \) is exactly a universal arrow from \( * \) to \( H \).

[p. 471. Lines 5–6.]

Abstraction consists in formulating essential aspects of some subject matter in terms of suitable axioms.
For ease of exposition, the presentation here is specialized to a familiar example for $J = [0, 1]$, namely the tent map defined by the function

$$f(x) = \begin{cases} 
2x & 0 \leq x < .5 \\
2 - 2x & .5 \leq x \leq 1 
\end{cases}$$
For any $y \in S_1$, with $y \neq 1$, there are exactly two values of $z$ (namely $z = y/2$ or $1 - y/2$) for which $\sigma(t_z) = t_y$. 


[p. 241. Lines 13–11 from bottom.]
A group is a set $G$ together with a binary operation $G \times G \to G$, written $(a, b) \mapsto ab$, such that: \ldots In other words, a group is a monoid in which every element is invertible.
Therefore these identities characterize the biproduct $A_1 \oplus A_2$ up to isomorphism . . .
The Preisach model of electromagnetic hysteresis dates from 1935 ... This model uses a superposition of especially simple independent relay hysteresis operators ... That is,

\[ F[v](t) = \int \int \mu(\alpha, \beta) F_{\alpha\beta}[v](t) \, d\alpha d\beta \]

where \(\mu(\alpha, \beta) \geq 0\) is a weight function, usually with support on a bounded set in the \((\alpha, \beta)\)-plane, \(F_{\alpha\beta}\) is a relay hysteresis operator with thresholds \(\alpha < \beta\), and

\[
\begin{align*}
h_U(v) &= +1 \quad \text{on } [\alpha, \infty); \\
h_L(v) &= -1 \quad \text{on } (-\infty, \beta].
\end{align*}
\]

[p. 3219. Last four lines of abstract.]

A characterization for all the smooth maps between the spaces of vector bundles, whose $k$th derivatives are linear differential operators of degree $r$ in each variable, is given in terms of $A^{(r)}$ maps.

[p. 530. Lines 10–9.]

In terms of this function, “bracket symbols” \( \left[ \begin{array}{c} n \\ j \end{array} \right] \) and “brace symbols” \( \left\{ \begin{array}{c} n \\ i \end{array} \right\} \) may be defined by the formulas . . .

[p. 236. Lines 4–8.]

... we use the weather outlook for the week as the basis for our world. We might write, for instance,

weather(sunday,fair).

Here we are representing the fact, “the weather on Sunday will be fair” by a predicate, weather, with two arguments.

[p. 739. Lines 23–25.]

Given two distinct points $a$ and $b$, the class of all points $p$ such that there exists a motion that leaves $a$ fixed and transforms $p$ into $b$ is called a *sphere* of center $a$ and passing through $b$. 
Citations


[p. 741. Lines 2–4.]

Notice that the rotation in Euclidean space described above not only fixes $a$ and $b$, but also fixes all points collinear with $a$ and $b$. 

[p. 116. Example 3.1.]

Let \( f : [0, 1] \to [0, 1] \) be defined by

\[
  f(x) = \begin{cases} 
    x + 0.5 & 0 \leq x \leq 0.5 \\
    0 & 0.5 \leq x \leq 1.
  \end{cases}
\]
Theorem. Let $x \in \mathbb{C}^N$ and let the values $p_{[\alpha]}(x)$ be nonnegative for all $\alpha \in \mathcal{A}$. Then necessarily all coordinates $x_i$, $i \in N$, are nonnegative.

[p. 575. Lines 3–5.]

These constraints are such that, if one player could in his turn leave (say) \( n \) matchsticks in a particular heap, then the other player could not. In particular, at most one of the players is entitled to clear any particular heap.
Citations


[p. 27. Line 4.]
Under function addition, \((M, +)\) is a group.
From now on we assume $n$ to be good, and we will freely identify elements of $H^1(G_K, E[n])$ with the $\Delta_n$-equivariant homomorphisms $G_M \to E[n]$ to which they give rise.

[p. 147. Lines 7–11.]

Perhaps the greatest advantage to the use of symbolic computation is that a student can work with relatively complicated examples without losing sight of the process for finding the solution. Such a method takes full advantage of the capacity of the software to substitute matrices into scalar polynomials, to perform integration, and to simplify matrix expressions.

[p. 994. Lines 9–11.]

Let $P$ denote the set of all ordered pairs of elements of $D$, and define the relation $R$ over $P$ by

$$(a, b)R(c, d) \iff ad = bc$$

[p. 547. Lines 17–18.]
Consequently, in order to evaluate $f(M, N)$ we need only evaluate $f(p^a, p^b)$ where $p$ is a prime.
Simultaneously, we define the *evaluation function*

\[ \text{ev} : \text{terms} \times \text{assignments} \rightarrow \text{values} \]

so that if \( \theta \) is a term and \( x \) is an assignment of elements from \( \omega \) and perhaps partial functions on \( \omega \), then \( \text{ev}(\theta, x) \) is the result of plugging \( x \) into \( \theta \).
290. (166, 200) Mckenzie, R. (1975), ‘On spectra, and
the negative solution of the decision problem for identities
having a finite nontrivial model’. Journal of Symbolic Logic,
volume 40, pages 186–196.

[p. 187. First sentence of fifth paragraph.]

We use Polish notation (sometimes with added parentheses
for clarity) to denote the terms build up from function symbols
and variables of a first order language.

[p. 608. Lines 19–21.]

In order to ensure that no undesirable clash of variables occurs in what follows, we assume that the variables of the language of \( K \) are listed without repetition as \( x, y, u_0, u_1, z_0, z_2, z_2, x_0, x_1, x_2, \ldots \).
With respect to integration, however, he is well aware of his rather unhappy position, that of knowing that some functions cannot be integrated (in finite terms) but of possessing no way of determining whether a particular function is in this class or not.
Consider the monomial symmetric function $\langle a_1, a_2, \ldots, a_k \rangle$ in $Q[x_1, x_2, \ldots, x_n]$ and let $t = \sum_{i=1}^{k} a_i$. Then there is a positive rational number $c$ and an element $B$ in $Q[p_1, p_2, \ldots, p_{i-1}]$ such that

$$\langle a_1, a_2, \ldots, a_k \rangle = (-1)^{k_1} cp + B.$$

[p. 917. Lines 1–4 of Section 2.1.]

Two of the four points must lie in the same subregion from the partition shown in Figure 3a, so $d_4 \leq \frac{1}{\sqrt{3}}$. If the upper bound is attained, then one point must lie at the center and the other one is a vertex of the triangle.

[p. 119. Lines 8–7 from bottom.]

If the vertices of the triangle are covered by two discs, one of the discs must cover two vertices, . . .
Citations

In this section two additional operations, namely subtraction and negation, will be allowed. For example the combination \((-3)\) involves negation but not subtraction.
Citations


[p. 732. Lines 5–8.]

Since the inverse images of hulls are hulls, the map \( Q \in \text{Prim}(\mathcal{B}) \rightarrow \phi^{-1}(Q) \in \text{Prim}(\mathcal{A}) \) is continuous. It remains to be shown that it is one to one. Suppose that \( N, Q \in \text{Prim}(\mathcal{B}) \) are such that \( \phi^{-1}(N) = \phi^{-1}(Q) \). Choose continuous irreducible representations \( \pi, \sigma \) of \( \mathcal{B} \) on Banach spaces \( X, Y \) such that \( N = \ker(\pi) \) and \( Q = \ker(\sigma) \).
Citations


[p. 86–87. Lines 3–1 from bottom.]

Definition of a functional. A functional is a function on $C_1$ to $C_2$ whenever $C_1$ is itself a class of functions of a finite number of numerical variables while $C_2$ is another class of functions of a finite number of numerical variables.

[p. 170. Lines 6–4 above Theorem 4.3.]

In the preceding initial value type theorem the distributions and measures were restricted to have support in $[0, \infty)$ by the growth of $e^{-\sigma x}$ for negative $x$ and large positive $\sigma$. 

[p. 714. Line 4 from bottom.]
The $c_\xi$’s inhabit distinct equivalence classes:
Consider $F(x) = ax^2 + bx + c$ ($a, b, c \in \mathbb{Z}$), $a \neq 0$, and suppose $|F(x)|$ is prime for all integers $0, 1, \ldots, l-1$. If $l \in \mathbb{N}$ is the smallest value such that $|F(l)|$ is composite, $|F(l)| = 1$, or $|F(l)| = |F(x)|$ for some $x = 0, 1, \ldots, l - 1$, then $F(x)$ is said to have prime-production length $l$. 

Citations


[p. 531. Definition 2.1.]
The coordinates of the line at angle $\theta$ to the $x$-axis are the oriented projections of a unit length from that line onto the axes, namely $(\cos \theta, \sin \theta)$. Hence $G(1, \mathbb{R}^2)$ is just the unit circle in $\mathbb{R}^2$. 

[p. 716. Line 1.]

The function $\sin x/x$ is endlessly fascinating.
[p. 720. Lines 12–15.]

Flip a fair coin repeatedly. Beginning with 0, add $1/2$ if the result is heads and subtract $1/2$ if the result is tails. On the next toss add or subtract $1/4$; on the next add or subtract $1/8$, and so on.
Citations


[p. 749. Title of Article.]

The Principal Axis Theorem Over Arbitrary Fields

[p. 751. Theorem 2.]

Let $F$ be a formally real pythagorean field. The following are equivalent:

(i) $F$ has the Principal Axis Property,
(ii) Every symmetric matrix over $F$ is diagonalizable over $F$, and
(iii) Every symmetric matrix over $F$ has an eigenvalue in $F$. 

**Superattracting Root Theorem.** Let $n \in \mathbb{N}$. The parameter $r$ satisfies $Q_n(r) = 0$ and $Q_j(r) \neq 0$ for $0 < j < n$ if and only if iteration of $f_r(x)$ has a superattracting periodic point of period $n$. 


[p. 875. Lines 10–11.]

Let \((X, \mathcal{B}, m)\) be a standard Lebesgue measure space, namely a measure space whose \(\sigma\)-algebra is countably generated and countably separate.

[p. 911. Lines 13–12 from bottom.]

Let $X, I$ be sets, $(Y_i)_{i \in I}$ a family of sets, and for each $i \in I$ let $f_i : X \to Y_i$. 

[p. 39. Lines 8–10, lines 2–1 from the bottom, and lines 1–2 on page 40.]

The question we have to consider is, what is the probability that if a natural number be taken at random its first significant digit will be $n$, its second $n'$, etc. . .

Our problem is thus reduced to the following:

We have a series of numbers between 1 and $i$, . . .
The support of $F$ is the smallest closed set outside which $F$ vanishes identically.
If \( \theta \) is rational in degrees, say \( \theta = 2\pi r \) for some rational number \( r \), then the only rational values of the trigonometric functions of \( \theta \) are as follows: \( \sin \theta, \cos \theta = 0, \pm \frac{1}{2}, \pm 1 \); \( \sec \theta, \csc \theta = \pm 1, \pm 2 \); \( \tan \theta, \cot \theta = 0, \pm 1 \).

[p. 83. Lemma 7.1.]

Any \( r + 1 \) linear forms in \( r \) indeterminates with rational coefficients are linearly dependent over the rationals.

[p. 212. Lines 15–12 from bottom.]

Such cardinals do exist; in particular, taking $\mathcal{R}$ to be the set of rational numbers, well-ordering the irrationals, and definition $P_\lambda$ to be a sequence of rationals converging to the $\lambda$th irrational, shows that $\aleph_0$ is such a cardinal.

[p. 70. Lines 9–6 from bottom.]

If we let $x$ be the length of the new fence which is to be aligned with the original 100 feet and proceed in the usual fashion to maximize the resulting expression for area, $x$ comes out to be $-25$, which is not permitted by the conditions of the problem.
With this convention on sides, the defining property of a module homomorphism $\phi : M \rightarrow N$ is that $(r \cdot x + x \cdot y)\phi = r \cdot (x)\phi + s \cdot (y)\phi$ for all $x, y \in M$ and $r, s \in R$. 


[p. 760. Lines 14–16.]

[p. 766. Lines 14–17.]

Definition. If $R$ is a commutative Noetherian ring and $P$ is a prime ideal of $R$, then $P$ is associated with the module $M$ provided $P = (0 : x)$ for some $x \in M$. We will call $M$ $P$-primary if the prime ideal $P$ is associated with $M$ and no other prime is.
Before proceeding further with the argument, let us prove (27).
The isoperimetric inequality (36) gives
\[ L^2 \geq 4\pi A + \alpha^2 A^2 > \alpha^2 A^2 \]
for simply connected domains with \( K \leq -\alpha^2 \), so that (77) holds in that case.
Again using the parameter $s$ of arc length along the curve, and denoting by $x$ the position vector on the curve, we have the unit tangent vector
\[ T = \frac{dx}{ds} \]
and the curvature $k$ defined by
\[ k = \left| \frac{dT}{ds} \right| = \left| \frac{d^2x}{ds^2} \right|. \]
Taking $x = 2$ in (6), we obtain $\psi_n(2) = 1 + 2^{n-1}$, so that
\begin{equation}
\rho_n < 2 \quad (n \geq 2)
\end{equation}
and we see that all roots of a reduced equation lie in the disk $|z| < 2$. 

[p. 624. Lines 14–16.]

Taking $x = 2$ in (6), we obtain $\psi_n(2) = 1 + 2^{n-1}$, so that
\begin{equation}
\rho_n < 2 \quad (n \geq 2)
\end{equation}
and we see that all roots of a reduced equation lie in the disk $|z| < 2$. 

322
Citations


For what values of $c$ (if any) is it true that for every finite family of disjoint finite sets $F_j$ with $\text{diam } F_j > 0$ there exist disjoint polygonal arcs $A_i$ such that $F_j \subseteq A_i$ and $\text{diam } A_i \leq c \text{ diam } F_j$ for all $j$?

[p. Abstract. First two sentences.]

In this paper we present a generalization of Oberon’s record type extensions. Our extension mechanism is orthogonally applicable to all the conventional data types found in Pascal-like languages.


**Theorem 4 (Van Rees).** Let \( c = AP[AD, BC] \) be an irreducible Appolonian cubic, let \( A' \) and \( B' \) be two arbitrary regular points on the cubic, and let \( C' = B' + O_1 \), \( D' = A' + O_1 \). Then the two Appolonian cubics \( AP[AD, BC] \) and \( AP[A'D', B'C'] \) are identical.

[p. 566. Lines 1–4.]

Courant in [1] gives Lebniz’ rule for finding the Nth derivative of the product of two functions, and then remarks that no such easily remembered law has been found for the repeated differentiation of the compound function $Y = F[U(X)]$. 
We put $F_{v,w} = S$, $E_{v,w} = \mathcal{R}_{X_{v,w}}$, and define $\phi_{v,w}$ as the composite:

$$F_{v,w} = S \hookrightarrow (Q \oplus \mathcal{R})_{X_{v,w}} \xrightarrow{\text{pr}_2} E_{v,w} = \mathcal{R}_{X_{v,w}}.$$
Therefore, we may deduce that
\[
F(\xi, z) = \frac{T(\xi, z) - S(\xi, z)}{\sqrt{S(\xi, z)T(\xi, z)}}.
\]

\[
\exp \left\{ \frac{1}{2\pi i} \int_a^b \log \left\| \frac{A(\mu) - \xi - \epsilon k(\mu)}{A(\mu) - \xi + \epsilon k(\mu)} \right\| \frac{d\mu}{\mu - z} \right\}
\]
and it is now clear that the roots of \( F(\xi, z) \) are the roots of the function
\[
W(\xi, z) = \frac{T(\xi, z) - S(\xi, z)}{\sqrt{S(\xi, z)T(\xi, z)}}.
\]
\[ x = \frac{ah}{b - a} \]

Then we substitute this value for \( x \) in the two foregoing equations of sect 7.4, obtaining…
This discrepancy was due to fewer computers being used on the project and some “down time” while code for the final stages of the algorithm was being written.
If $n$ is not a square modulo $p$, then $Q(x)$ is never divisible by $p$ and no further computations with $p$ need be done.

[p. 4. Line 5.]

... where \( j \) denotes the imaginary unit.

[p. 264. Line 5.]

All algebras in this section will have a unit denoted by 1.

[p. 490. Lines 4–8.]

3. **Constrained optimization and the Maratos effect.** In this section we consider the problem of calculating the least value of \( \{ F(x) ; x \in \mathbb{R}^n \} \) subject to the constraints

\[
(3.1) \quad c_j(x) = 0, \quad i = 1, 2, \ldots, m,
\]

on the variables. Usually there are some inequality constraints too, but the purpose of this section is achieved without this extra complication.
About 15 years ago, E. Becker gave a talk in which he proved that

\[ B(t) := \frac{1 + t^2}{2 + t^2} \in \mathbb{Q}(t) \]

is a sum of $2n$-th powers of elements in $\mathbb{Q}(t)$ for all $n$.

... A rational function $f \in \mathbb{R}(X) := \mathbb{R}(x_1, \ldots, x_k)$ is positive semi-definite (psd) if $f \geq 0$ at every point in $\mathbb{R}^k$ for which it is defined.
In effect, we thereby provide an elementary operational semantics for our program flowgraphs, saying that $L(F)$ is the “meaning” of $F$, describing as it does all possible sequences of elementary processes that could result. We note, however, that this is truly an “elementary” semantics, far less detailed than the denotational semantics that one ordinarily introduces in a programming language context.
Addition, subtraction, multiplication, division, exponentiation, or taking of roots, form an apparently tidy set of operations, but it is a set with some enticing anomalies.

[p. 321. Lines 6–8 of abstract.]

When $1 \leq p < q \leq 2$ or $2 \leq q < p \leq \infty$ it is known that

\[ L^p_p \subset L^q_q. \]

The main result of this paper is that the inclusion in (1) is strict unless $G$ is finite.
The first time I gave the course, I asked the students not to submit their first drafts. But their attitude towards written homework was so ingrained that they would not even read their work, let alone revise it.
If we adjoin to the above condition the further clause:

\[(Ex)(y)(z) \ (J(y, z) \in W_x \equiv J(y, z) \notin W_i)\]

then the definition becomes a definition of the class of recursive well-orderings (or, rather, of the corresponding set of indices), for this clause just says that the predicate $W^2_i$ has an r.e. complement $\overline{W}^2_x$, and a predicate is recursive just in case it and its complement are both r.e.

[p. 521. Lines 6–10.]

One formula implies another if there is no assignment of truth values which makes the first formula true and the second false. Two formulas are equivalent if they imply each other. Implication and equivalence, so defined, are relations of formulas; they are not to be confused with the conditional and biconditional, commonly expressed by $\supset$ and $\leftrightarrow$. 

[p. 168. Lines 10 and 3–1 from bottom.]

Let \( \{x\} \) denote the fractional part of \( x \), that is, \( \{x\} = x - \lfloor x \rfloor \).

... 2. If \( k \) is an integer, \( a \) is a real number in the range \( 1 < a < 2 \), and \( x = k/(a - 1) \), then

\[
\left\lfloor a\{x\} + \frac{a}{2} \right\rfloor = \lfloor ax \rfloor
\]

At the other extreme, Figure 3 shows a worst case degenerate tree where each node has only 1 child except for the single leaf. [The trees here are binary trees.]

[p. 509. Lines 1–2.]

If $\Upsilon_1$ and $\Upsilon_2$ are topologies defined on the set $X$, then $\Upsilon$ is the \textit{intersection topology} w.r.t. $\Upsilon_1$ and $\Upsilon_2$ defined on $X$, where $\{U_1 \cap U_2 \mid U_1 \in \Upsilon_1 \text{ and } U_2 \in \Upsilon_2\}$ is a basis for $\Upsilon$. 

344
For any $RG$-module $M$ of $R$-rank $m$, there is an exact sequence
\[ 0 \to U \otimes_R M \to RG \oplus \cdots \oplus RG \to \bar{R}^G \otimes_R \bar{M} \to 0, \]
where $m$ summands occur in the center module. This implies by Schanuel’s Lemma that the module $U \otimes_R M$ depends only upon $\bar{M}$. 

Citations


[p. 272. Lines 7–4.]

[p. 391. Lines 12–13.]

Suppose that there is a non-trivial solution to Fermat’s equation $X^\ell + Y^\ell = Z^\ell$. 

346

[p. 529. Lines 8–9.]

I may also consider the sequence of all proper powers, which includes the 5th powers, 7th powers, 11th powers, etc.
For $\alpha$ irrational, this defines a sequence $p_0/q_0, p_1/q_1, p_2/q_2, \ldots$ of best approximations to $\alpha$, with $q_0 < q_1 < q_2 < \ldots$. In fact, an initial segment of the sequence can be calculated by trial and error from the definition simply by considering increasing $q$. 

[p. 475. Lines 7–8 below the figure.]

To state the problem precisely, suppose that \( y = g(x) \) is a piecewise smooth nonnegative curve defined over \([a, b]\), and is revolved around the \( x \)-axis.

[p. 265. Lines 15–12.]

The system of counting numbers consists of the elements 1, 2, 3, ⋯, and the fundamental operations (addition, subtraction, multiplication and division) for combining them.
If we take variables and put them on either side of “=” or “<”, we obtain what we shall call atomic formulas. E.g.,

\[ x = w, \ y < y, \ x_4 < z_2 \]

are atomic formulas. These are the basic formulas (i.e., expressions) from which we shall build the larger formulas of our language.

[p. 342. Lines 9–11.]

The theorem is the following: *the set of all sums of rearrangements of a given series of complex numbers is the empty set, a single point, a line in the complex plane, or the whole complex plane.*
We sometimes refer to a function as a map or mapping and say that $f$ maps $S$ into $T$. 

Citations

Citations


[p. 1440. .]

The family of all partitions of a set (also called equivalence relations) is a lattice when partitions are ordered by refinement.

[p. 12. First two lines of Example 4.2.]
Consider the degenerate triangle $G(p)$ in $\mathbb{R}^2$ shown in Fig. 4 with collinear vertices...
Citations


[p. 421. Lines 9–8 from bottom.]

We are now in a position to give a conceptual proof of Pringsheim’s theorem . . .
1.2 Definition: A \textit{k-coloring} (or \textit{proper k-coloring}) of a graph is an assignment of \(k\) colors to the vertices of the graph in such a way that no two adjacent vertices receive the same color.
The equations of motion for $w$, when accompanied with the change of independent variables $ds = dt/r(t)$ introduced by Sundman, not only are well defined at $w = 0$, a collision, but they assume the particularly simple form

$$u'' + au = 0$$

where $a$ is some positive constant and $u = (u_1, u_2)$ is the vector representation of $w$. 
(2) the set $Z(f')$ is contained within the interval defined by the extreme points of $Z(f)$. A similar constraint holds for $Z(f'')$ with respect to the zeros of $f'$. 


[p. 413. Lines 1–3.]

Given an instance of a mathematical structure, we are led to ask for its group of symmetries; i.e., the group of structure-preserving self-transformations.

[p. 114. Line 9.]

Then \( s_i = (\lambda, \ldots, \lambda, r, \lambda, \ldots, \lambda) \), where \( r \) inhabits the \( i \)th position.

[p. 141. Title.]

Another Proof of Chebysheff’s Inequality

[p. 890. Beginning of Section 1.]

**What exactly is a Moebius strip?** On one hand, it is often defined as the topological space attained by starting with a (closed) rectangle, endowed with the “usual” topology, and identifying two opposite edges point by point with each other, so that each vertex gets identified with the one diagonally across. This “abstract Moebius strip” serves in topology as the canonical example of a nonorientable manifold.

On the other hand, there is a physical model of the abstract strip, and it is usually denoted by the same term . . .

[p. 408. Lines 5–4 from bottom.]

As string theory developed, a very rich mathematical structure emerged, but one which now has very little resemblance to strong interaction.
Let $H$ be a finite subgroup of $S0(3)$ of order $n$. To each element of $H$ (other than the identity) there corresponds an axis that intersects the sphere in two points.

[p. 515. Lines 4–5.]

All functions considered in this paper are real valued functions of a single real variable.
We define the terms by the generalized inductive definition:
i) a variable is a term; ii) If $u_1, \ldots, u_n$ are terms and $f$ is $n$-ary, the $fu_1, \ldots, u_n$ is a term.
If we have an element $u$ of the free Lie algebra $L$ and write $u = u(x_1, \ldots, x_n)$, this just means that no generators $x_i$ with $i > n$ are involved in $u$. 

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Citations


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[pp. 155. Lines 18–16 from bottom.]

The main theorem states that if $G$ is of type I (resp. II), then the larger group $G$ admits no free, $k$-cohomologically trivial action on any space $X \sim_2 (S^n)^d$, for any $n$ (resp. for any $n$ not of the form $2^f \cdot 5^e - 1$).

[p. 366. Line 15–14 from the bottom.]

For example, when \( n = 2 \), the products above have the Polish form \( XXabc \) and \( XaXbc \) and the reverse Polish forms \( abXcX \) and \( abcXX \).
We factor $\Phi_n(b)$ in the ring of algebraic integers of $Q_n = Q(\zeta)$. Then

$$\Phi_n(b) = \prod_{\substack{a=1 \leq a \leq n \cr (a,n)=1}} (b - \zeta^a) \quad (1)$$

We now claim that if $A$ is the ideal in $R$ generated by two distinct factors $\zeta - a^{a_1}$ and $\zeta - a^{a_2}$ given in (1), then . . .
... we let $B(x, \xi) = \sqrt{P_m(x, \xi)}$ and notice that we can factor the symbol of the operator in (1.5) as follows

$$P_m(x, \xi) - \tau^m = (B(x, \xi) - \tau) \cdot (B^{m-1} + B^{m-2} \tau + \cdots + \tau^{m-1})$$

The second factor is uniformly elliptic in the sense that it is bounded below by a multiple of $(|\xi|^{m-1} + \tau^{m-1})$, while the first factor vanishes for certain $|\xi| \approx \tau$. 

376.
A set is a strict subset of a set \( B \), written \( A \subset B \), if and only if \( A \subseteq B \) and \( A \neq B \).

[p. 31. Last three lines.]

**(1.3) Theorem.** An immersed circle in $S^2$ and its tantrix share a regular homotopy class. A tantrix in the equator’s class always bounds oriented area $2\pi \pmod{4\pi}$. A trantrix in the other class bounds zero.
The function $\chi: g \to \text{Trace} (\rho(g))$ of $G$ into $\mathbb{C}$ is called the character of $\rho$. 

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[p. 774. Problem 5746.]

\[ S(a) = \sum_{x,y,z} e \left\{ x + y + z + \frac{a}{yz + zx + xy} \right\} \]

[p. 884. Problem E 2198.]

If $r > 1$ is an integer and $x$ is real, define

$$f(x) = \sum_{k=0}^{\infty} \sum_{j=1}^{r-1} \left\lfloor \frac{x + jr^k}{r^{k+1}} \right\rfloor,$$

where the brackets denote the greatest integer function.
An elementary number theory problem is to determine the possible forms of squares among the positive integers. For instance, it is easy to see that any square must be of the form $3k$ or $3k + 1$. 

We shall denote by **WCH** Jones’ hypothesis that \(2^{\aleph_0} < 2^{\aleph_1}\), since it is a weak version of **CH**: If \(2^{\aleph_0} = \aleph_1\), then \(2^{\aleph_0} = \aleph_1 < 2^{\aleph_1}\) by Cantor’s theorem. Clearly the consistency of **CH** implies the consistency of **WCH**. The negation of **WCH**, namely \(2^{\aleph_0} = 2^{\aleph_1}\), is called the Luzin Hypothesis (**LH**) \ldots
Theorem. If $y_1 = y_1(x)$ and $y_2 = y_2(x)$ are functions satisfying $0 \leq y_1(0) < y_2(0)$ and the differential equations

$$\frac{dy_1}{dx} = Cy_2^\alpha, \quad \frac{dy_2}{dx} = Cy_1^\alpha \quad (4.4)$$

where $C > 0$, then for some constant $c_0$

$$y_2(x) - y_1(x) \rightarrow \begin{cases} 
0 & \alpha > 0 \\
c_0 & \alpha = 0 \\
\infty & \alpha < 0
\end{cases} \quad (4.5)$$

as $x$ increases without limit in the (possibly infinite) domain of definition of $y_1(x)$ and $y_2(x)$. 


[p. 2614. Lines 17–11 from bottom.]
... then any inequality

$$f(x_1, \ldots, x_n) \geq 0,$$

where the function $F$ is formed by any finite number of rational operations and real exponentiations, is decidably true or false!

[p. 251. Lines 9–11.] We begin by minimizing not $|x|$ but $\frac{1}{2} |x|^2 = \frac{1}{2} x^T x$, and we introduce the constraint $Ax = b$ through Lagrange multipliers; there are $m$ equations in the constraint, and therefore $m$ multipliers $\lambda_1, \ldots, \lambda_m$. 
Certainly the expression $e^T C e = c_1 e_1^2 + \cdots + c_4 e_4^2$ is not negative. It is zero only if $e = Ax = 0$. 


[p. 107. Lines 15–16.]

Certainly the expression $e^T C e = c_1 e_1^2 + \cdots + c_4 e_4^2$ is not negative. It is zero only if $e = Ax = 0$. 

387
Citations

391. (115, 259) Straight, H. J. (1993), *Combinatorics: An Invitation*. Brooks/Cole. [p. 17. 3.] Given sets $X$ and $Y$, a function from $X$ to $Y$ is a subset $f$ of $X \times Y$ with the property that, for every $x \in X$, there is a unique $y \in Y$ such that $(x, y) \in f$. 
Citations

Let \( \{a_n\} \) be an increasing sequence of positive integers such that \( \log a_n/q_n \to 0 \) as \( n \to \infty \), where \( q_n \) is the least prime factor of \( n \).

\[ \text{Citations} \]


[p. 728. Lines 14–15.]
Citations


[p. 321. Abstract.]

We consider the Schrödinger equation

\[- \frac{d^2}{dx^2} \psi + \epsilon (\cos x + \cos (\alpha x + \vartheta)) \psi = E \psi\]

where ...

[p. 644. Lines 14–8.]

It is this requirement of being able to uniformly partition the interval that limits the scope of the Riemann integral. It would be much more desirable to somehow allow “variable length” partitions; for example, if one were attempting to approximate the area under the graph of \( f(x) = \frac{1}{\sqrt{x}}, 0 < x \leq 1 \), it would be natural to take the subintervals in an approximating partition to be very fine near the singularity \( x = 0 \).

[p. 257. Abstract.]

Let $(\Omega, \Sigma, P)$ be a complete probability space. Let $(\Sigma_j)_{j \in J}$ be a directed family of sub-$\sigma$-algebras of $\Sigma$. Let $(\Phi, \Psi)$ be a pair of conjugate Young functions.
For simplicity, let us say that a Banach space contains a copy of $L^1$ if it contains a subspace isomorphic to $L^1$. 

[p. 494. Lines 1–4.]

... may find a homeomorphism $x : E \to \mathbb{P}^1_k$ such that

$$x(\gamma u) = \frac{ax(u) + b}{cx(u) + d}.$$  

We will tend to abuse notation and identify $E$ with $\mathbb{P}^1_k$ by means of the function $x$. 

399
In particular, he derived Kepler’s laws of motion from the assumption that the sun pulls on a planet with a force that varies inversely with the square of the distance from the sun to the planet.

[p. 518. Line before (4.11).]

Plugging in we obtain:

[p. 730. Third line from the bottom.]

\[\left| \sin \left( \frac{2\pi}{a} \left( \frac{a}{4} + t \right) \right) - \sin \left( \frac{2\pi}{a} \cdot \frac{a}{4} \right) \right| < \sin \frac{2\pi}{a} p\]
A function $f$ whose domain of definition is $X$, and whose range is $Y$ is frequently denoted by $f : X \rightarrow Y$, and is referred to as a function on $X$ onto $Y$. 

Eventually a good definition of tangent was devised. Succinctly put,

the tangent is the limit of the secant.

Let us formalize this definition. I shall give a formal definition, not of a tangent to a curve, but of a tangent to a set of points.
Citations


... (the symbols $A_2, A_3, \cdots, G_2$ have their usual meaning, and the left superscript denotes the degree of $\tilde{k}/k$ when $\tilde{k} \neq k$, i.e, when $G$ does not split over $k$)

if $k = F_2$, groups of type $A_2, 2A_3, 2A_4, B_3$ and $3D_4$;
if $k = F_3$, groups of type $A_2, 2A_2, 2A_3, B_2$ and $3D_4$ and $G_2$. 

405

[p. 749. Line 9.]

In this paper we analyze the mathematical structure of the black-oil flow equations.

[p. 1. Lines 1–2.]

Let \( u = A_1x^n + A_2x^{n-1} + A_3x^{n-2} + \ldots + A_{n-1} = \psi x \ldots \)

The expression \( \psi x \) is read “function of \( x \)”.

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407

[p. 525. Lines 17–20.]

The example also illustrates the point that it is not always easy to decide whether two equations in more than two unknowns are consistent, quite aside from the matter of producing in that case a real solution.
Citations


[p. 35. Lines 8–4 from bottom.]

... We shall show that a larger matching exists. (We mean larger in *cardinality*; we may not be able to find a complete matching containing these particular $m$ edges.)

[p. 435. Theorem A.]

If $A$ is a closed subspace of the normal space $X$ then there is a function $\eta : C^*(A) \to C^*(A)$ such that for every $f \in C^*(A)$, $\eta(F)$ extends $F$ and has the same bounds as $F$. 

410
For example, each of the properties of being a group, an Abelian group, or a torsion-free Abelian group is expressible in the so-called elementary language (or first-order predicate calculus). Thus, instead of saying that the group $G$ is Abelian, we can say it is a model of the elementary sentence $\forall x \forall y (x \circ y = y \circ x)$. Such properties are also called elementary.

... the existence of such an $\mathfrak{A}$ is of basic importance in the remarkable work of Gödel and Cohen on the consistency and independence of the continuum hypothesis and other basic propositions in set theory.

[p. 25. Lines 12–11, 6–2 from bottom.]

$T$ is called $\omega$-complete if whenever $T \vdash \forall v_0 (\theta_n(v_0) \rightarrow \phi(v_0))$ for all $n$, then $T \vdash \forall v_0 \phi(v_0)$ . . . It is worthwhile noting that in contrapositive form the definition of $\omega$-complete reads:

(3) $T$ is $\omega$-complete if and only if for any 1-formula $\phi$, if $T + \exists v_0 \phi(v_0)$ has a model, then for some $n$, $T \exists v_0 (\theta_n(v_0) \land \phi(v_0))$ has a model.
This may be written as

\[ \sum_{j=4}^{7} \left( \sum_{i} b_i a_{ij} \right) \cdot \left( \sum_{k} a_{jk} c_k^q - \frac{e_j^{q+1}}{q+1} \right) = 0 \]

by invoking (15) to imply that the first bracket is zero for \( j = 2, 3 \). Since the second bracket is zero for \( 4 \leq j \leq 6 \) by \((17'')\), and ...

[p. 786. Lines 8–7 from bottom.]

If $f, g \in \mathcal{P}(V_0 \times V_0^*)$ then let $\{f, g\}$ (the Poisson bracket of $f$ and $g$) be as in Appendix 1.
Suppose \( n \) is an odd prime. In modern terms, Gauss has shown that the field generated over the rationals by the \( n \)-th roots of unity contains \( \sqrt{\pm n} \); here we must take the plus sign when \( n \) is of the form \( 4k + 1 \) and the minus sign when \( n \) is of the form \( 4k + 3 \).

[p. 465. Lines 1–10.]

First, we plan to show how properties of analytic functions of a complex variable can be used to obtain several results of classical harmonic analysis (that is, the theory of Fourier series and integrals of one real variable). This will be done in Section 1. Second, in Section 2 we shall indicate how some of these applications of the theory of functions can be extended to Fourier analysis of functions of several variables.

Some of these applications of the theory of functions seem very startling since the results obtained appear to involve only the theory of functions of a *real* variable or the theory of measure.

[p. 15. Lines 1–7.]

To determine the global structure of a complex we need to have first a knowledge of the local structure, of behaviour in a neighborhood of a point; with this understanding of local structure, we can solve the problem of recognition of a 2-complex by putting the pieces together, as it were. In the case of graphs, the problem of local structure is completely resolved by a knowledge of the degree of a vertex. The matter is less simple for surfaces.


Now by the method of “conformal representation” of the theory of functions the pressure exerted on a plane wing of various shapes, by the motion of the air, may in some cases be calculated, and the center of pressure may also be found.
Therefore, if $n > 1$ we have

$$Q(n) = \frac{1}{n!} \left\{ \sum_{i=1}^{n} (n - 1)! + \sum_{\substack{i,j=1 \atop i \neq j}}^{n} (n - 2)! \right\}$$

What sort of creature now inhabits the curly braces?

[p. 682. Lines 9–5 from bottom.]

Constraint C0 is really a way of saying we wish to maximise $z$ where

$$z \leq -4x_1 + 5x_2 + 3x.$$ 

By maximising $z$ we will “drive” it up to the maximum value of the objective function. It would clearly be possible to treat C0 as an equation but for simplicity of exposition we are treating all constraints as $\leq$ inequalities.
To translate the question into more precise mathematical language, we consider a grid of $MN$ lattice points

$$G = \{(i, j) \mid 0 \leq i \leq M - 1; 0 \leq j \leq N - 1\}$$

and we regard them as being the vertices of a graph.
By composing \( \tilde{f} \) with the inverse of \( \sigma \), we may assume the restriction of \( \tilde{f} \) to \( \mathbb{Z}^n \) is the identity. Because

\[(**) \mathbb{R}/\mathbb{Z} \text{ is compact},\]

this implies that \( \tilde{f} \) moves points by a bounded amount . . .

[p. 146. Lines 2–3.]
The right hand side of (5.36) equals $\frac{2\epsilon}{\lambda}(1 + h\lambda)^k - 1$ [sic], which is always less than or equal to $\frac{2\epsilon}{\lambda}e^{\lambda T}$. 

424

A complex function $\eta$ defined on $[0, 1]$ still has exactly one best approximation from $\mathcal{P}_n(\mathbb{C})$. . . . If the function being approximated is the real function $f$, then its best approximation in $\mathcal{P}_n(\mathbb{C})$ is also real . . .

[p. 44. Lines 14–13 from bottom.]
In our formalism we adopt prefix notation in preference to the infix/postfix notation used by Girard . . .

[p. 656. Last line.]

\[
\left((x + 1)^{\nu_1} - 1\right)\left((x + 1)^{\nu_1} + 1\right) = x^z
\]

[p. 486. Lines 4–6.]

Feller (1937) showed that if normal convergence occurs (that is, condition (2.2) holds), but condition (2.4) also obtains, then

\[
\frac{1}{\rho \, s_{n_k}} \frac{X_{m_k}}{s_{n_k}} \Rightarrow (0, 1).
\]

[p. 813. Lines 4–3 from the bottom.]

Here $f^s$ is the spherical derivative of the function; the present notation . . . is better adapted for displaying the argument of the function explicitly.
Accordingly, let
\[ \hat{f}(\xi, \eta) = \int \int f(x, y) e^{i(\xi x + \eta y)} \, dx \, dy \]
be the Fourier transform of \( f \). We shall need only two facts about \( \hat{f} \): it is continuous, and the correspondence between \( f \) and \( \hat{f} \) is one-to-one. In particular, if \( \hat{f} = 0 \) then \( f = 0 \) (almost everywhere).

[p. 102. Lines 1–2 of Abstract.]

Let \((X, Y, \nu)\) be the volume space formed as the product of the volume spaces \((X_i, Y_i, \nu_i)\) \((i = 1, 2)\).
Let us return for a moment to the circle $S^1 \subseteq \mathbb{C} = \mathbb{R}^2$.

**Missing Links**

- mental representation – conceptual blending
- elementary – informal jargon
- well-defined – mod
- name – names from other languages
- pronunciation – names from other languages