Citations

Page number(s) in parentheses show the pages of this Handbook on which the quotation is referenced.

1. (28, 229, 249) **Abate, M. (1997)**, 'When is a linear operator diagonalizable?'. *American Mathematical Monthly*, volume **104**, pages 824–830.

[p. 827. Lines 5–7.]

Let $\mathbf{c} = (c_0, \ldots, c_s) \in r^{s+1}$ be a finite sequence of real numbers. If $c_0 \ldots c_s \neq 0$, the *number of variations in sign* of $\mathbf{c}$ is the number of indices $1 \leq j \leq s$ such that $c_{j-1}c_j < 0$ (that is, such that c_{j-1} and c_j have opposite signs.)

2. (198) **Adem, A. (1997)**, 'Recent developments in the cohomology of finite groups'. *Notices of the American Mathematical Society*, volume , pages 806–812.

[p. 806. Lines 20–22, second column.]

Now taking generators for IG as a $\mathbb{Z}G$-module, we can map a free $\mathbb{Z}G$-module of finite rank onto IG.

3. (160) **Adler, A. and S.-Y. R. Li (1977)**, 'Magic cubes and Prouhet sequences'. *American Mathematical Monthly*, volume **84**, pages 618–627.

[p. 619. Lines 12–13.]

Definitions. By an *N-cube of order T*, we mean an N-dimensional array of T^N N-dimensional boxes.

4. (178, 269) **Aigner, M. (1995)**, 'Turán's graph theorem'. *American Mathematical Monthly*, volume **102**, pages 808–816.

[p. 813. Lines 12–13.]

Now, we associate to each $i \in V$ a variable x_i (over $\mathbb{R}$) and consider the function $f(x_1, \ldots, x_n) = 2\sum_{ij \in E} x_i x_j$.

5. (99, 252) **Akin, E. and M. Davis (1985)**, 'Bulgarian solitaire'. *American Mathematical Monthly*, volume **92**, pages 237–250.

[p. 243. Line 3.]

We have already proved (a) and the formal proof of (b) is very similar.

6. (131) **Aldous, D. (1994)**, 'Triangulating the circle, at random'. *American Mathematical Monthly*, volume **101**, pages 223–233.

[p. 232. Lines 4–2 from bottom.]

How do you actually draw a 2000-edge tree ... which "looks right", and what metric space do such trees inhabit?

7. (19, 135, 261) **Aldous, D. and P. Diaconis (1986)**, 'Shuffling cards and stopping times'. *American Mathematical Monthly*, volume **93**, pages 333–348.

[p. 333. Lines 3–2 above Figure 1.]

By an inductive argument, all $(n-1)!$ arrangements of the lower cards are equally likely.

8. (226, 236, 248) **Alfred, U. (1964)**, 'Consecutive integers whose sum of squares is a perfect square'. *Mathematics Magazine*, volume **37**, pages 19–32.

[p. 19. Lines 11–8.]

Substituting into the above formula, we obtain

$$5x^2 + 20x + 30 = z^2.$$

Since 5 is a factor of all terms on the left, it must be a factor of z.

9. (21, 33, 198, 222) **Alperin, J. L., R. Brauer, and D. Gorenstein (1970)**, 'Finite groups with quasi-dihedral and wreathed Sylow 2-subgroups'. *Transactions of the American Mathematical Society*, volume **151**, pages 1–261.

[p. 16. Proposition 3 and lines 6 and 11–12 of the proof.]

Proposition 3. If G is a QD-group with quasi-dihedral Sylow 2-subgroups whose unique nonsolvable composition factor satisfies the First Main Theorem, then for any involution x of G and any Sylow 2-subgroup of S of $C(x)$, we have $[O(C(x))] \subseteq O(G)$. ... Let L be the simple normal QD-subgroup of G. ... Clearly $L \cap O(N) = L \cap O(C(x)) \subseteq O(C_l(x))$. But by assumption L satisfies the conclusion of the First Main Theorem ...

10. (170, 204, 238) **Anderson, B. (1993)**, 'Polynomial root dragging'. *American Mathematical Monthly*, volume **100**, pages 864–866.

[p. 864. Lines 8–3 from bottom.]

Let $p(x)$ be a polynomial of degree n with all real distinct roots $x_1 < x_2 < \cdots < x_n$. Suppose we "drag to the right" some or all of these roots. I.e. we construct a new nth degree polynomial q with all real distinct roots $x'_1 < x'_2 < \cdots < x'_n$ such that $x'_i > x_i$ for all integers i between 1 and n. The derivatives of p and q, which of course are polynomials of degree $n-1$, must also have all real distinct roots from Rolle's theorem.

11. (227) **Antman, S. S. (1980)**, 'The equations for large vibrations of strings'. *American Mathematical Monthly*, volume **87**, pages 359–370.

[p. 364. Formula 3.10..]

Let us set

$$\eta(x,t) = \phi(x)\psi(t)\mathbf{e}$$

12. (142, 188, 203, 252) **Anton, H. (1984)**, *Elementary Linear Algebra, Fourth Edition.* John Wiley and Sons.

[p. 121. Lines 10–13.]

Proof. By hypothesis, the coefficients a, b, and c are not all zero. Assume, for the moment, that $a \neq 0$. Then the equation $ax+by+cz+d=0$ can be rewritten as $a(x+(d/a))+by+cz=0$. But this is a point-normal form of the plane passing through the point $(-d/a, 0, 0)$ and having $\mathbf{n}=(a,b,c)$ as a normal.

13. (209) **Apostol, T. M. (1969)**, *Calculus, Volume II, Second Edition*. Blaisdell Publishing Company.

[p. 93. Lines 1–8.]

Using Theorem 3.9 we express det A in terms of its kth-row cofactors by the formula

$$\det A = \sum_{j=1}^{n} a_{kj} \operatorname{cof} a_{kj} \tag{3.30}$$

Keep k fixed and apply this relation to a new matrix B whose ith row is equal to the kth row of A for some $i \neq k$, and whose remaining rows are the same as those of A. Then $\det B = 0$ because the ith and kth rows of B are equal. Expressing det B in terms of its ith-row cofactors we have

$$\det B = \sum_{i=1}^{n} b_{ij} \operatorname{cof} b_{ij} = 0. \tag{3.31}$$

14. (24, 67, 118) **Applebaum, C. H. (1971)**, 'ω-homomorphisms and ω-groups'. *Journal of Symbolic Logic*, volume **36**, pages 55–65.

[p. 56. Lines 9–8 from the bottom.]

DEFINITION. The ω-group G_1 is ω-homomorphic to the ω-group G_2 [written: $G - 1 \geq_\omega G_2$] if there is at least one ω-homomorphism from G_1 onto G_2.

15. (20) **Arbib, M. A. and E. G. Manes (1974)**, ‘Machines in a category: An expository introduction’. *SIAM Review*, volume **16**, pages 163–192.

[p. 169. Lines 1–4.]

A group may be thought of as a set with 3 operators, a binary operation labeled $\cdot$ (we say the label $\cdot$ has *arity* 2 since $\cdot$ labels a 2-*ary* operator); a unary operation labeled ^{1} (which has arity 1), and a constant labeled e (we say e has arity 0, and refer to constants as nullary operators).

16. (98, 131, 178) **Arveson, W. (1974)**, 'Operator algebras and invariant subspaces'. *The Annals of Mathematics, 2nd Ser.*, volume **100**, pages 433–532.

[p. 464. Lines 6–5 from bottom.]

Now by the preceding paragraph we can find an m-null set $M \subseteq X$ such that, for all $x \in M$, $|\mu_X|$ lives in G_x.

17. (264) **Atiyah, M. F. (1976)**, 'Bakerian lecture, 1975: Global geometry'. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, volume **347**, pages 291–299.

[p. 293. Lines 15–17.]

Suppose now we move on to an equation with two unknowns, for example

$$x^2 + y^2 = 1, \qquad (1)$$
$$x^3 + y^2 = 1. \qquad (2)$$

18. (188, 251) **Atkinson, M. P., L. Daynes, M. J. Jordan, T. Printezis, and S. Spence (1996)**, 'An orthogonally persistent Java'. *ACM SIGMOD Record*, volume **25**, pages 68–75.

[p. Abstract. Third through fifth sentences.]

Here we report on a particular approach to providing such facilities, called "orthogonal persistence". Persistence allows data to have lifetimes that vary from transient to (the best approximation we can achieve to) indefinite. It is orthogonal persistence if the available lifetimes are the same for all kinds of data.

19. (76, 261) **Axler, S. (1995)**, 'Down with determinants!'. *American Mathematical Monthly*, volume **102**, pages 139–154.

[p. 142. Proposition 3.4.]

The generalized eigenvectors of T span V.

20. (198, 245, 275) **Axler, S. (1995)**, 'Down with determinants!'. *American Mathematical Monthly*, volume **102**, pages 139–154.

[p. 153. Lines 8–7 from bottom.]

Thus the function A defined on $\operatorname{ran}\sqrt{S^*S}$ by $A(\sqrt{S^*S}u) = Su$ is well defined and is a linear isometry from $\operatorname{ran}\sqrt{S^*S}$ onto $\operatorname{ran} S$.

21. (24, 67) **Baer, R. (1955)**, 'Supersoluble groups'. *Proceedings of the American Mathematical Society*, volume **6**, pages 16–32.

[p. 16. Lines 18–19.]

DEFINITION. *The group G is supersoluble if every homomorphic image $H \neq 1$ of G contains a cyclic normal subgroup different from* 1.

22. (116) **Baker, H. H., A. K. Dewdney, and A. L. Szilard (1974)**, 'Generating the nine-point graphs'. *Mathematics of Computation*, volume **28**, pages 833–838.

[p. 835. Lines 4–3 above Figure 2.]

In Fig. 2(a) below, a nine-point graph is shown.

23. (15, 250, 261) **Balinski, M. L. and H. P. Young (1977)**, 'Apportionment schemes and the quota method'. *American Mathematical Monthly*, volume **84**, pages 450.

[p. 451. Theorem 1.]

An apportionment method M is house-monotone and consistent if and only if it is a Huntington method.

24. (51, 130, 238) **Banuelos, R. and C. N. Moore (1989)**, 'Sharp estimates for the nontangential maximal function and the Lusin area function in Lipschitz domains'. *Transactions of the American Mathematical Society*, volume **312**, pages 641–662.

[p. 641. Lines 11–8.]

We call $\phi : \mathbf{R}^n \to \mathbf{R}$ a Lipschitz function if there exists a constant M such that $|\phi(x) - \phi(y)| \leq M\,|x - y|$ for all $x, y \in \mathbf{R}^n$. The smallest such M for which this inequality remains valid for all x and y will be called the Lipschitz constant of ϕ.

25. (148) **Barnes, C. W. (1984)**, 'Euler's constant and e'. *American Mathematical Monthly*, volume **91**, pages 428–430.

[p. 429. Lines 10–11.]

We follow the customary approach in elementary calculus courses by using the definition $\ln(x) = \int_1^x t^{-1}\,dt$.

26. (78, 117, 276) **Bartle, R. C. (1996)**, 'Return to the Riemann integral'. *American Mathematical Monthly*, volume **103**, pages 625–632.

[p. 626. Lines 17–19 and 36.]

Usually the partition is ordered and the intervals are specified by their end points; thus $I_i := [x_{i-1}, x_i]$, where

$$a = x_0 < x_1 < \cdots < x_{i-1} < x_i < \cdots < x_n = b.$$

... A strictly positive function δ on I is called a *gauge* on I.

27. (25, 134, 198) **Bartle, R. C. (1996)**, 'Return to the Riemann integral'. *American Mathematical Monthly*, volume **103**, pages 625–632.

[p. 627. Lines 23–24.]

(3.2) If $h : [0,1] \to \mathbf{R}$ is Dirichlet's function (= the characteristic function of the rational numbers in $[0,1]$), then $h \in \mathcal{R}^*([0,1])$ and $\int_0^1 h = 0$.

28. (38, 107, 120, 125, 194, 227, 235, 250, 261) **Bartle, R. C. (1996)**, 'Return to the Riemann integral'. *American Mathematical Monthly*, volume **103**, pages 625–632.

[p. 631. Lines 3–6, 13–14, 19.]

(8.2) Dominated Convergence Theorem. *Let* (f_n) *be a sequence in* $\mathcal{R}^*([a,b])$, *let* g, $h \in \mathcal{R}^*([a,b])$ *be such that*

$$g(x) \leq f_n(x) \leq h(x) \quad \textit{for all} \quad x \in [a,b],$$

and let $f(x) = \lim_n f_n(x) \in \boldsymbol{R}$ *for all* $x \in [a,b]$. *Then* $f \in \mathcal{R}^*([a,b])$ *and (8a) holds.*

... As usual, we define a *null set* in $I := [a,b]$ to be a set that can be covered by a countable union of intervals with arbitrarily small total length.

... *Every* $f \in \mathcal{R}^*(I)$ *is measurable on* I.

29. (34, 211) **Bass, H. and R. Kulkarni (1990)**, 'Uniform tree lattices'. *Journal of the American Mathematical Society*, volume **3**, pages 843–902.

[p. 845. Lines 22–23.]

We put $i(e) = [\mathcal{A}_{\partial_0 e} : \alpha_e \mathcal{A}_e]$ and call $(A, i) = I(\mathbf{A})$ the corresponding *edge-indexed graph.*

30. (250) **Bateman, P. T. and H. G. Diamond (1996)**, ‘A hundred years as prime numbers’. *American Mathematical Monthly*, volume **103**, pages 729–741.

[p. 729. Lines 3–6.]

The theorem is an asymptotic formula for the counting functions of primes $\pi(x) := \#\{p \leq x : p \text{ prime}\}$ asserting that

$$\pi(x) \sim x/(\log x)$$

The twiddle notation is shorthand for the statement $\lim_{x\to\infty} \pi(x)/\{x/\log x\} = 1$.

31. (7) **Bateman, P. T. and H. G. Diamond (1996)**, 'A hundred years as prime numbers'. *American Mathematical Monthly*, volume **103**, pages 729–741.

[p. 738. Lines 12–10 from bottom.]

In 1937 A. Beurling introduced an abstraction of prime number theory in which multiplicative structure was preserved but the additive structure of integers was dropped.

32. (200) **Bauer, F. L. (1977)**, 'Angstl's mechanism for checking wellformedness of parenthesis-free formulae'. *Mathematics of Computation*, volume **31**, pages 318–320.

[p. 318. Just below the first figure.]

The formula in parenthesis-free (polish) form [2] is now written over the fixed bars . . .

33. (106, 181, 193) **Bauer, H. (1978)**, 'Approximation and abstract boundaries'. *American Mathematical Monthly*, volume **85**, pages 632–647.

[p. 644. Lines 4–2 above the figure.]

We obtain

$$\Phi(X) = \big\{\big(x, u(x)\big) \mid x \in [a, b]\big\}$$

which is the graph of the function u.

34. (253, 277) **Baudisch, A. (1996)**, 'A new uncountably categorical group'. *Transactions of the American Mathematical Society*, volume **348**, pages 3889–3940.

[p. 3902. Lines 20–24.]

Therefore we can assume w.l.o.g. that $r^i_{s_1 t_1} = 0$ for $2 \leq i \leq l$. Using the linear independence of $\Phi_1, \ldots, \Phi_l$ over $\wedge^2 A$ we iterate this procedure. Therefore we can assume w.l.o.g. that there are pairwise distinct pairs (s_i, t_i) such that $r^i_{s_1 t_1} \neq 0$ and $r^j_{s_i t_i} = 0$ for $i \neq j$.

35. (56) **Baveye, P. and G. Sposito (1984)**, 'The operational significance of the continuum hypothesis in the theory of water movement through soils and aquifers'. *Water Resources Research*, volume **20**, pages 521–530.

[p. 521. Title.]

The operational significance of the continuum hypothesis in the theory of water movement through soils and aquifers

36. (122, 193, 253) **Beardon, A. F. (1996)**, 'Sums of powers of integers'. *American Mathematical Monthly*, volume **103**, pages 201–213.

[p. 201. Lines 1–2.]

Our starting point is the well known identity

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$

37. (122, 255) **Beesack, P. R. (1971)**, ‘Integral inequalities involving a function and its derivative’. *American Mathematical Monthly*, volume **78**, pages 705–741.

[p. 705. Lines 12–14.]

In fact it turns out that *any* nontrivial admissible F_2 (that is, $F_2 \in C^1[0,1]$ with $F_2(0) = F_2(1) = 0$ but $F_2(x) \not\equiv 0$) satisfies the second of the above inequalities.

38. (265) **Beeson, M. J. (1975)**, 'The nonderivability in intuitionistic formal systems of theorems on the continuity of effective operations'. *Journal of Symbolic Logic*, volume **40**, pages 321–346.

[p. 332. Lines 3–1 from bottom.]

The idea of the proof is to unwind the proof of MS_1 given in §1.3 into a direct diagonal construction, and modify that construction so as to obtain the extra information needed to realize the conclusion from the realizing functionals of the hypothesis.

39. (91) **Beeson, M. (1976)**, 'The unprovability in intuitionistic formal systems of the continuity of effective operations on the reals'. *Journal of Symbolic Logic*, volume **41**, pages 18–24.

[p. 19. Lines 8–10.]

It is quite simple to reduce the problem to showing the existence of a certain "pathological" function defined (not necessarily on all reals but) on indices provable in R.

40. (25, 64, 67, 68, 120) **Bellamy, D. P. (1975)**, 'Weak chainability of pseudocones'. *Proceedings of the American Mathematical Society*, volume **48**, pages 476–478.

[p. 476. Lines 1–4.]

A continuum is a compact metric space. $I = [0, 1]$; $A = (0, 1]$; S is the unit circle in the complex numbers. If X is a continuum, a *pseudocone over* X is a compactification of A with remainder X.

41. (79, 99, 276) **Bell, H., J. R. Blum, J. V. Lewis, and J. Rosenblatt (1966)**, *Modern University Calculus with Coordinate Geometry.* Holden-Day, Inc.

[p. 48. Proof.]

Proof. Let N be the set of those positive integers which satisfy the following conditions:
(a) 1 is a member of N,
(b) whenever x is a member of N, then $x \geq 1$.
We need to show that N is precisely the set of all positive integers to prove our result.

42. (78, 211, 267) **Belna, C. L., M. J. Evans, and P. D. Humke (1979)**, 'Symmetric and strong differentiation'. *American Mathematical Monthly*, volume **86**, pages 121–123.

[p. 121. Line 1.]

Throughout we let f denote a real valued function defined on the real line $\mathbf{R}$.

43. (84, 143, 264, 268) **Bender, E. A. (1987)**, 'The number of three-dimensional convex polyhedra'. *American Mathematical Monthly*, volume **94**, pages 7–21.

[p. 14. Lines 7–14.]

Complete the square in (3.2) to obtain an equation of the form

$$(F + \text{stuff})^2 = G\left(x, y, z, F_1\right). \qquad (3.3)$$

Let $z = Z(x, y)$ stand for the value of z for which the left side of (3.3) vanishes. Since the left side of (3.3) is a square, its derivative with respect to z also vanishes at Z. Applying this to the right side of (3.3) we obtain the two equations

$$G(x, y, Z, F_1) = 0 \quad \text{and} \quad G_z(x, y, Z, F_1) = 0$$

in the two unknowns Z and F_1. These are rational equations, and they can be "solved" for F_1.

44. (32, 120, 171) **Bendersky, M. and D. M. Davis (1994)**, '3-primary v_1-periodic homotopy groups of F_4 and E_6'. *Transactions of the American Mathematical Society*, volume **344**, pages 291–306.

[p. 295. Lines 5–4 from bottom.]

The Toda bracket $\langle -, 3, \alpha_1 \rangle$ is essentially multiplication by v_1, which acts nontrivially from $v_{4i+2}S^{23}$ to $v_{4i+6}S^{23}$.

45. (107, 110, 139, 252, 266) **Bezem, M. (1989)**, 'Compact and majorizable functionals of finite type'. *Journal of Symbolic Logic*, volume **54**, pages 271–280.

[p. 271. Lines 8–6 from bottom.]

We shall occasionally use lambda-notation to specify functionals, ie $\lambda X.$— specifies a functional F such that $FX =$ — for all X.

46. (140, 141, 261) **Bhatia, R. and P. Šemrl (1997)**, 'Approximate isometries on Euclidean spaces'. *American Mathematical Monthly*, volume **104**, pages 497–504.

[p. 502. Lemma 3.]

Lemma 3. *Let f and g be as in Lemma 2, and let $u \in E_n$ be a unit vector. Then for every $x \in E_n$ orthogonal to u we have $|\langle f(x), g(u)\rangle| \leq 3\varepsilon$.*

47. (185) **Bhatia, R. and P. Šemrl (1997)**, 'Approximate isometries on Euclidean spaces'. *American Mathematical Monthly*, volume **104**, pages 497–504.

[p. 503. Line 9 from bottom.]

Since f is an ε-isometry, we have

$$m - \varepsilon < \|f(x + my) - f(x)\| < m + \varepsilon,$$

or equivalently,

$$m - \varepsilon < \|(m - a + b_m)y + u_m\| < m + \varepsilon.$$

48. (45, 117, 219) **Bieri, R. and J. R. J. Groves (1986)**, 'A rigidity property for the set of all characters induced by valuations'. *Transactions of the American Mathematical Society*, volume **294**, pages 425–434.

[p. 425. Abstract.]

We prove that $\Delta(G)$ satisfies a certain rigidity property and apply this to give a new and conceptual proof of the Brewster-Roseblade result [**4**] on the group of automorphisms stabilizing G.

49. (192, 204, 231) **Bivens, I. and A. Simoson (1998)**, 'Beholding a rotating beacon'. *Mathematics Magazine*, volume **71**, pages 83–104.

[p. 93. Lines 12–8 from bottom.]

The singular solution of Clairaut's equation is the envelope of this family of lines and can be parametrized by the pair of equations $x = -f'(t)$ and $y = -tf'(t) + f(t)$. On the other hand, we could consider a one-parameter family $\mathcal{F}$ of lines $y = f(b)x + b$ in which the parameter is the *y-intercept* of each line.

50. (117, 117, 124, 189) **Billingsley, P. (1973)**, 'Prime numbers and Brownian motion'. *American Mathematical Monthly*, volume **80**, pages 1099–1115.

[p. 1107. Lines 3–2 from bottom.]

For an illustration of this theorem, suppose the A in (10) is the set $[x : \alpha \leq x(1) \leq \beta]$ of paths in $C_0[0, 1]$ that over the point $t = 1$ have a height between α and β.

51. (139) **Birkhoff, G. D. (1908)**, 'Boundary value and expansion problems of ordinary linear differential equations'. *Transactions of the American Mathematical Society*, volume **9**, pages 373–395.

[p. 389. Lines 10–12.]

In both (46) and (47) it is to be noted that the Σ-terms are not important except at $x = a$ and $x = b$, since the real parts of the exponential terms are large and negative for l large and $a < x < b$.

52. (98) **Birman, J. S. (1993)**, 'New points of view in knot theory'. *Bulletin of the American Mathematical Society (N.S.)*, volume **28**, pages 253–287.

[p. 279. Lines 23–24.]

Using Lemma 2, we find a closed braid representative K_β of BK, $\beta \in B_n$.

53. (142) **Bivens, I. and A. Simoson (1998)**, 'Beholding a rotating beacon'. *Mathematics Magazine*, volume **71**, pages 83–104.

[p. 96. Lines 12–13 and 15.]

THEOREM 4. *Let S denote the boundary point of the ordinary set. Then S is either on the screen or on the envelope of the family of separation lines.*

Proof. ... Suppose S is a boundary point of the set of ordinary points that does not belong to the screen.

54. (59, 181, 261) **Blass, A. (1979)**, ‘Natural endomorphisms of Burnside rings’. *Transactions of the American Mathematical Society*, volume **253**, pages 121–137.

[p. 122. Lines 19–20.]

For any two G-sets X and Y, there are obvious actions of G on the disjoint union $X + Y$ and (componentwise) on the product $X \times Y$.

55. (67, 69, 133) **Blecksmith, R., M. McCallum, and J. L. Selfridge (1998)**, '3-smooth representations of integers'. *American Mathematical Monthly*, volume , pages 529–543.

[p. 529. Lines 6–7 of Introduction.]

A 3-*smooth number* is a positive integer whose prime divisors are only 2 or 3.

56. (61, 61, 142, 194) **Blecksmith, R., M. McCallum, and J. L. Selfridge (1998)**, '3-smooth representations of integers'. *American Mathematical Monthly*, volume , pages 529–543.

[p. 535. Lines 10–9 from bottom.]

Corollary. Assume that $n > 1$ is prime to 6. Then n has a unique representation if and only if the 2-rep and the 3-rep of n agree.

57. (101, 253) **Bogart, K. P. (1989)**, 'A fresh(man) treatment of determinants'. *American Mathematical Monthly*, volume **96**, pages 915–920.

[p. 915. Lines 10–8 from bottom.]

In order to give a quick treatment for the n by n case, it is tempting to give a known formula for the determinant, such as the permutation or the column expansion formula, ...

58. (147) **Bolis, T. S. (1980)**, 'Degenerate critical points'. *Mathematics Magazine*, volume **53**, pages 294–299.
[p. 294. Lines 4–9.]

The basic tool for determining the local behavior of a function at a point of its domain is Taylor's theorem. The problem whose solution we pursue is the following: *Given a point a in the domain of a sufficiently smooth function f, what is the least degree of a Taylor polynomial of f at a that adequately describes the local behavior of f at a?*

59. (61, 252, 261) **Borwein, D., J. M. Borwein, P. B. Borwein, and R. Girgensohn (1996)**, ‘Giuga’s conjecture on primality’. *American Mathematical Monthly*, volume **103**, pages 40–50.

[p. 41. Lines 4–2.]

So, for example, no Carmichael number has the prime factors 3 and 7 at the same time. This property was used by Giuga to prove computationally that each counterexample has at least 1000 digits.

60. (33, 41, 147) **Borwein, J. M., P. B. Borwein, and D. H. Bailey (1989)**, ‘Ramanujan, modular equations, and approximations to pi or how to compute one billion digits of pi’. *American Mathematical Monthly*, volume **96**, pages 201–129.

[p. 204. Lines 12–15.]

In part we perhaps settle for computing digits of π because there is little else we can currently do. We would be amiss, however, if we did not emphasize that the extended precision calculation of pi has substantial application as a test of the ”global integrity ” of a supercomputer.

61. (251, 268) **Boudreau, P. E., J. J. S. Griffin, and M. Kac (1962)**, 'An elementary queueing problem'. *American Mathematical Monthly*, volume **69**, pages 713–724.

[p. 717. Lines 19–21.]

But the analyticity of $G(z, w)$ requires that the numerator of the fraction vanish whenever the denominator does.

62. (251) **Boyce, W. E. and R. C. DiPrima (1965)**, *Elementary differential equations*. Wiley.

[p. 66. Lines 13–16.]

The stated conditions imply that the mass of the vehicle is given by

$$m(t) = m_0 - \beta t \qquad t < t_1$$

At the time t_1, the mass of the vehicle has been reduced to the value $m_1 = m_0 - \beta t_1 = m_0 - m_f$.

63. (107, 124, 221) **Bredon, G. E. (1971)**, 'Counterexamples on the rank of a manifold'. *Proceedings of the American Mathematical Society*, volume **27**, pages 592–594.

[p. 592. Lines 3–5.]

The Poincaré polynomial of M is $P_M(t) = \sum b_i t^i$ where b_i is the ith Betti number of M. Thus rank $M \geq 1$ iff -1 is a root of $P_M(t)$.

64. (255) **Bridges, D. S. (1977)**, ‘A constructive look at orthonormal bases in Hilbert space’. *American Mathematical Monthly*, volume **84**, pages 189–191.

[p. 189. End of second paragraph.]

We also consider the trivial space $\{0\}$ to be of finite dimension 0.

65. (110, 147, 240) **Britt, J. (1985)**, 'The anatomy of low dimensional stable singularities'. *American Mathematical Monthly*, volume **92**, pages 183–201.

[p. 184. Lines 7–9.]

The mapping behaves very differently at singular points from the way it does at regular ones, where locally it behaves as if it were a mapping onto its codomain.

66. (8, 25, 31, 79, 98, 228, 249, 259, 261) **Brickman, L. (1993)**, 'The symmetry principle for Möbius transformations'. *American Mathematical Monthly*, volume **100**, pages 781–782.

[p. 782. Lemma 2.]

Lemma 2. *For each circle or extended line E, there is a unique $\bar{T} \in \bar{\mathcal{M}}$ such that*

$$E = \{z \in \hat{\mathbf{C}} : \bar{T}(z) = z\}.$$

(E is exactly the set of fixed points of $\bar{T}$.) This $\bar{T}$ is an involution of $\hat{\mathbf{C}}$; that is, $\bar{T} \circ \bar{T}$ is the identity.

67. (86, 120, 130, 135) **Brillhart, J. and P. Morton (1996)**, 'A case study in mathematical research: The Golay-Rudin-Shapiro sequence'. *American Mathematical Monthly*, volume **103**, pages 854–869.

[p. 863. Last four lines.]

We get the inequality we want as long as

$$\frac{\sqrt{6n}+2^{k+1}}{\sqrt{n+2^{2k+1}}}<\sqrt{6},$$

i.e., as long as $\sqrt{6n}+2^{k+1} < \sqrt{6n+3\cdot 2^{2k+2}}$. The latter inequality is equivalent to $n < 2^{2k+1}/3$, which is true in the interval we're considering ($n \leq M_{k-1}$).

68. (59, 85, 120, 204, 258) **Brink, C. and J. Pretorius (1992)**, 'Boolean circulants, groups, and relation algebras'. *American Mathematical Monthly*, volume **99**, pages 146–152.

[p. 146. Lines 18–22.]

Let $\mathcal{B}_n$ be the algebra which has as the base set all n-square Boolean matrices and is endowed with the componentwise Boolean operations of complementation $'$, meet $\cdot$ and join $+$ (under which it forms a Boolean algebra) as well as the matrix operations of transposition and multiplication ; , and the identity matrix I.

69. (47, 76, 141, 250) **Bruce, J. W. (1993)**, 'A really trivial proof of the Lucas-Lehmer test'. *American Mathematical Monthly*, volume **100**, pages 370–371.

[p. 370. Theorem 1.]

Theorem 1 (LUCAS-LEHMER). Let p be a prime number. Then $M_p = 2^p - 1$ is a prime if M_p divides S_{p-1}.

70. (99, 124, 198) **Bruckner, A. M., J. Marik, and C. E. Weil (1002)**, 'Some aspects of products of derivatives'. *American Mathematical Monthly*, volume **99**, pages 134–145.

[p. 140. Lines 2–3.]

As an illustration of this theorem let us consider a function u with the following properties …

71. (22, 261) **Bryant, V. (1993)**, *Aspects of Combinatorics.* Cambridge University Press.

[p. 62. Lines 10–18.]

Theorem For any integer $n > 1$ there exist at most $n - 1$ mutually orthogonal $n \times n$ Latin squares.

72. (184, 252) **Buckholtz, D. (1997)**, 'Inverting the difference of Hilbert space projections'. *American Mathematical Monthly*, volume **104**, pages 60–61.

[p. 60. Lines 1–3.]

Let R and K be subspaces of a Hilbert space H, and let P_R and P_K denote the orthogonal projections of H onto these subspaces. When is the operator $P_R - P_K$ invertible? ...

73. (48, 48) **Buhler, J., D. Eisenbud, R. Graham, and C. Wright (1994)**, 'Juggling drops and descents'. *American Mathematical Monthly*, volume **101**, pages 507–519.

[p. 513. Lines 8–9.]

Which finite sequences correspond to juggling patterns? Certainly a necessary condition is that the average must be an integer. However this isn't sufficient.

74. (98) **Bumby, R. T., F. Kochman, and D. B. West, editors (1993a)**, ‘Problems and solutions’. *American Mathematical Monthly*, volume **100**, pages 796–809.

[p. 796. Problem 10331.]

Find all positive integers n such that $n!$ is multiply perfect; i.e., a divisor of the sum of its positive divisors.

75. (29, 168, 184, 238) **Bumby, R. T., F. Kochman, and D. B. West, editors (1993b)**, 'Problems and solutions'. *American Mathematical Monthly*, volume **100**, pages 498–505.

[p. 499. Problem 10311.]

It is well-known that if g is a primitive root modulo p, where $p > 2$ is prime, either g or $g + p$ (or both) is a primitive root modulo p^2 (indeed modulo p^k for all $k \geq 1$.)

76. (77, 274) **Burrow, M. D. (1973)**, 'The minimal polynomial of a linear transformation'. *American Mathematical Monthly*, volume **80**, pages 1129–1131.

[p. 1129. Lines 4–6.]

It seems that none of the textbooks on linear algebra gives a direct proof of the fact that the minimal polynomial $m(x)$ of a linear transformation T is of degree less than or equal to the dimension of the vector space V on which T acts. The usual proof depends on the fact that $m(x)$ divides the characteristic polynomial $f(x)$, the degree of which equals the dimension of V, and for this one needs the Cayley-Hamilton Theorem.

77. (10, 98, 277) **Burgstahler, S. (1986)**, 'An algorithm for solving polynomial equations'. *American Mathematical Monthly*, volume **93**, pages 421–430.

[p. 423. Lines 7–11.]

The new algorithm can now be described. To approximate roots of $P(x) = 0$ (which, without loss of generality, is assumed not to have a root at $x = 0$):

Step 1: If the desired root is known to be near the origin, solve $P(1/z) = 0$ for $z = 1/x$.

Step 2: Determine a preliminary root estimate $R \neq 0$.

Step 3: Use R and formula (6) to find x_i and replace R by this number.

Step 4: If R is unsatisfactory as a root estimate, repeat step 3. (Or repeat step 1 if $|R| \ll 1$.)

78. (115, 133, 219, 259, 268) **Burton, D. M. (1994)**, *Elementary Number Theory, Third Edition.* Wm C. Brown Publishers. [p. 17. Theorem 2–1.]

79. (47, 61, 76, 250, 262) **Burton, D. M. (1994)**, *Elementary Number Theory, Third Edition.* Wm C. Brown Publishers. [p. 24. Corollary 2.]

80. (268) **Burckel, R. B. (1997)**, 'Three secrets about harmonic functions'. *American Mathematical Monthly*, volume **104**, pages 52–56.

[p. 55. Lines 4–6.]

In fact, this result is part of a large subject called quadrature problems that interested readers can find more about in . . .

81. (17, 85) **Burger, T., P. Gritzmann, and V. Klee (1996)**, 'Polytope projection and projection polytopes'. *American Mathematical Monthly*, volume **103**, pages 742–755.

[p. 744. Lines 18–20.]

Our general setting is Euclidean n-space $\mathbb{R}^n$ (with $n > 2$), equipped with the standard inner product $\langle \cdot, \cdot \rangle$ and the induced norm $\| \cdot \|$.

82. (8) **Busenberg, S., D. C. Fisher, and M. Martelli (1989)**, 'Minimal periods of discrete and smooth orbits'. *American Mathematical Monthly*, volume **96**, pages 5–17.

[p. 8. Lines 2–4.]

Therefore, a normed linear space is really a pair $(\mathbf{E}, \|\cdot\|)$ where $\mathbf{E}$ is a linear vector space and $\|\cdot\| : \ \mathbf{E} \to (0, \infty)$ is a norm. In speaking of normed spaces, we will frequently abuse this notation and write $\mathbf{E}$ instead of the pair $(\mathbf{E}, \|\cdot\|)$.

83. (126) **Butzer, P. L. and R. L. Stens (1992)**, 'Sampling theory for not necessarily band-limited functions: A historical overview'. *SIAM Review*, volume **34**, pages 40–53.

[p. 44. Lines 3–1 from bottom.]

Since the series in (3.2) generally has an infinite number of terms and the discrete parameter m of (1.1) or (1.2) has been replaced by a continuous $W > 0$, a different notation is used in the following;...

84. (108, 147, 181) **Callahan, J. (1974)**, 'Singularities and plane maps'. *American Mathematical Monthly*, volume **81**, pages 211–240.

[p. 217. Lines 1–6.]

Until now we have discussed the shape of a function on its whole domain, even though it is useful and perhaps more natural to work locally. For example, it is not difficult to show that $y = x^3 - 3x$ has folds at the points $x = \pm 1$ (i.e., is equivalent to $y = x^2$ in a neighborhood of each point) and in a neighborhood of every other point is a homeomorphism. On the other hand, this function is globally equivalent to neither a homeomorphism nor a (single) fold.

85. (84, 214) **Call, G. S. and D. J. Velleman (1993)**, 'Pascal's matrices'. *American Mathematical Monthly*, volume **100**, pages 372–376.

[p. 373. Theorem 2.]

Theorem 2. *For any real numbers x and y, $P[x]P[y] = P[x+y]$.*

86. Call, G. S. and D. J. Velleman (1993), 'Pascal's matrices'. *American Mathematical Monthly*, volume **100**, pages 372–376.

[p. 375. Theorem 5.]

Theorem 5. *For every real number x, $P[x] = e^{xL}$.*

87. (180, 268) **Carmichael, R. D. (1932)**, 'Some recent researches in the theory of numbers'. *American Mathematical Monthly*, volume **39**, pages 139–160.

[p. 142. Lines 6–9.]

We shall entirely pass over several of Ramanujan's contributions to the theory of numbers, including his investigation of the expression of integers in the form $ax^2 + by^2 + cz^2 + dt^2$ and his theory of highly composite numbers ...

88. (117, 117) **Carr, D. M. (1982)**, 'The minimal normal filter on $p_\kappa\lambda$'. *Proceedings of the American Mathematical Society*, volume **86**, pages 316–320.

[p. 316. Lines 2–3.]

Unless specified otherwise, κ denotes an uncountable regular cardinal and λ is a cardinal $\geq \kappa$.

89. (269) **Charron, R. J. and M. Hu (1995)**, 'a-contractivity of linearly implicit multistep methods'. *SIAM Journal on Numerical Analysis*, volume **32**, pages 285–295.

[p. 285. Lines 12–10 from bottom.]

In this paper we therefore focus on the absolute contractivity properties of linearly implicit methods (i.e., $b_k^{(i)} = 0$ for $i = 1, \ldots, s-1$) with fixed and variable step length.

90. (24, 34, 93, 100, 120, 211, 234) **Chew, J. (1988)**, 'A note on closure continuity'. *American Mathematical Monthly*, volume **95**, pages 744–745.

[p. 744. Lines 2–6.]

A function $f : X \to Y$ is closure continuous at $x \in X$ provided that if V is an open set in Y containing $f(x)$, there exists an open set U in X containing x such that $f(\bar{U}) \subset \bar{V}$. (Here context makes in which topological spaces the closure operations are taken; i.e. the bar on U refers to closure in X while the bar on V refers to closure in Y.)

91. (25, 29, 184, 227, 261) **Chow, T. Y. (1999)**, 'What is a closed-form number?'. *American Mathematical Monthly*, volume **106**, pages 440–448.

[p. 444. Lines 12–14.]

Definition. A *tower* is a finite sequence $A = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ of nonzero complex numbers such that for all $i \in \{1, 2, \ldots n\}$ there exists some integer $m_i > 0$ such that $\alpha_i^{m_i} \in A_{i-1}$ or $e^{\alpha_i m_i} \in A_{i-1}$ (or both).

92. (249, 251) **Christensen, C. (1996)**, ‘Newton’s method for resolving affected equations’. *The College Mathematics Journal*, volume **27**, pages 330–340.

[p. 330. Lines 9–13.]

The debate over priority for Newton’s method may now be settled, but almost forgotten in the discussion is that Newton presented his method for approximating real roots side by side with a similar method for writing y in terms of x when y is implicitly defined in terms of x by a polynomial equation – a so-called “affected equation”.

93. (19, 106, 107, 242) **Church, A. (1942)**, 'Differentials'. *American Mathematical Monthly*, volume **49**, pages 389–392.

[p. 390. Footnote.]

A variable is *free* in a given expression (in which it occurs) if the meaning or value of the expression depends upon determination of a value of the variable; in other words, if the expression can be considered as representing a function with that variable as argument. In the contrary case, the variable is called a *bound* (or *apparent*, or *dummy*) variable.

94. (138, 178, 252) **Clifford, A. H. (1959)**, 'Connected ordered topological semigroups with idempotent endpoints II'. *Transactions of the American Mathematical Society*, volume **91**, pages 193–208.

[p. 106. Lines 4–1 from bottom.]

The binary operation in S, which we now denote by $\circ$, is completely determined by that in T, which we denote by juxtaposition, and the mappings ψ and θ as follows (wherein $x, y \in T^0$; $t \in T$; $\kappa, \lambda \in K$):

95. (134) **Conkwright, N. B. (1934)**, 'The method of undetermined coefficients'. *American Mathematical Monthly*, volume **41**, pages 228–232.

[p. 228. Lines 1–4.]

In introductory courses in differential equations a certain process, the so-called method of undetermined coefficients, is often employed to find a particular integral of the linear differential equation with constant coefficients

$$f(D)y = \left(D^n + a_1 D^{n-1} + \cdots + a_{n-1} D + a_n\right) y = r(x). \tag{1}$$

96. (47, 252) **Copper, M. (1993)**, 'Graph theory and the game of Sprouts'. *American Mathematical Monthly*, volume **100**, pages 478–482.

[p. 480. Lemma 1.]

Suppose that the cubic graph G arises as just described from a complete game of Sprouts played on m vertices in p plays. Then

$$f = 2 + p - m$$

97. (120, 238) **Coven, E. M., G. A. Hedlund, and F. Rhodes (1979)**, 'The commuting block maps problem'. *Transactions of the American Mathematical Society*, volume **249**, pages 113–138.

[p. 116. Lines 14–15.]

We will use the power notation to denote repeated composition. Formally, $f^0 = I$ and for $n \geq 0$, $f^{n+1} = f \circ f^n$.

98. (10, 276) **Cull, P. and J. E. F. Ecklund (1095)**, 'Towers of Hanoi and analysis of algorithms'. *American Mathematical Monthly*, volume **92**, pages 407–420.

[p. 408. Lines 14–20.]

We would like to say that one algorithm is faster, uses less time, than another algorithm if when we run the two algorithms on a computer the faster one will finish first. Unfortunately, to make this a fair test, we would have to keep a number of conditions constant. For example, we would have to code the two algorithms in the same programming language . . .

99. (89) **Culler, M. and P. B. Shalen (1992)**, 'Paradoxical decompositions, 2-generator Kleinian groups, and volumes of hyperbolic 3-manifolds'. *Journal of the American Mathematical Society*, volume **5**, pages 231–288.

[p. 235. Lines 7–6 from bottom.]

We establish some notation and conventions that will be used throughout the paper.

100. (116) **Curjel, C. R. (1990)**, 'Understanding vector fields'. *American Mathematical Monthly*, volume **97**, pages 524–527.

[p. 524. Lines 17–16 from bottom.]

In the following exercises students have to use ruler and pencil to work on graphs and curves given by drawings.

101. (34, 64, 73, 118) **Currie, J. (1993)**, 'Open problems in pattern avoidance'. *American Mathematical Monthly*, volume **100**, pages 790–793.

[p. 790. Lines 8–3 from bottom.]

A **word** is a finite sequence of elements of some finite set Σ. We call the set Σ the **alphabet**, the elements of Σ **letters**. The set of all words over Σ is written Σ^*.

... The **empty word**, with no letters, is denoted by ε.

102. (116) **Dankner, A. (1978)**, 'On Smale's axiom A dynamical systems'. *Annals of Mathematics*, volume **107**, pages 517–553.

[p. 539. 5.2.7.]

On $\{r_4 \leq r \leq r_5 \text{ and } z \leq z_2\}$ the action of f on z-coordinates is multiplication by $e(r)$, where $e(r)$ is a smooth bump function with graph shown in Figure 4.

103. (72, 117, 193, 232) **Darst, R. and C. Goffman (1970)**, 'A Borel set which contains no rectangles'. *American Mathematical Monthly*, volume **77**, pages 728–729.

[p. 729. Line 23.]

Then $\phi(0) = m_1([U \cap F] \cap [U \cap G]) > .8\epsilon$.

104. (37, 133, 171, 175, 185, 211, 261, 280) **Davey, B. A. and H. A. Priestley (1990)**, *Introduction to Lattices and Order*. Cambridge University Press.

[p. 3. Beginning of 1.4.]

Each of $\mathbb{N}$ (the natural numbers $\{1, 2, 3, \ldots\}$), $\mathbb{Z}$ (the integers) and $\mathbb{Q}$ (the rational numbers) also has a natural order making it a chain.

105. (17, 171) **Dayton, B. H. and C. A. Weibel (1980)**, 'K-theory of hyperplanes'. *Transactions of the American Mathematical Society*, volume **257**, pages 119–141.

[p. 139. Lines 12–14.]

R. K. Dennis has shown that $-\mathrm{dlog}\,\{u,v\} = (uv)^{-1}\,du \wedge dv$ implies that -dlog $(\langle a,b\rangle) = (1+ab)^{-1}\,da \wedge db$ for all pointy brackets $\langle a,b\rangle$.

106. (117, 269) **de Boor, C. and K. Höllig (1991)**, 'Box-spline tilings'. *American Mathematical Monthly*, volume **98**, pages 793–802.

[p. 795. Line 6 from bottom.]

In this situation, it is convenient to introduce the new variables

$$(u, v) := \Xi^T x = (\xi^T x, \eta^T x).$$

107. (50, 117) **DeTemple, D. W. (1993)**, 'A quicker convergence to Euler's constant'. *American Mathematical Monthly*, volume **100**, pages 468–470.

[p. 468. Lines 1–4.]

Euler's constant γ is usually defined by the limit relation

$$\gamma = \lim_{n\to\infty} D_n,$$

where

$$D_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log n.$$

108. (52, 79, 261) **Devaney, R. L. and M. B. Durkin (1991)**, 'The exploding exponential and other chaotic bursts in complex dynamics'. *American Mathematical Monthly*, volume **98**, pages 217–233.

[p. 222. Lines 10–9 from bottom.]

That is, if a point is contained in $J(F)$, then so are all of its images and all of its preimages.

109. (142, 178) **Ž. Djoković, D. (1982)**, 'Closures of conjugacy classes in classical real linear Lie groups. II'. *Transactions of the American Mathematical Society*, volume **270**, pages 217–252.

[p. 233. Lines 1 and 8.]

Let first $k = 0$, i.e, $m = n$. … Now let $k > 0$. …

110. (79, 124) **Dornhoff, L. L. and F. E. Hohn (1978)**, *Applied Modern Algebra*. Macmillan.

[p. 166. Lines 1–3.]

A monoid $[M, \circ]$ with identity element 1 is a *group* iff for each $m \in M$ there is an inverse element $m^{-1} \in M$ such that

$$m^{-1} \circ m = m \circ m^{-1} = 1$$

111. (32, 59) **Downey, R. and J. F. Knight (1992)**, 'Orderings with αth jump degree 0^{α}'. *Proceedings of the American Mathematical Society*, volume **114**, pages 545–552.

[p. 546. Lemma 1.1.]

If C is r.e. in X but not recursive in X, then there is an ordering $\mathbf{A}$ such that $\mathbf{A}$ is recursive in C and no copy of $\mathbf{A}$ is recursive in C.

112. (39, 194) **Drasin, D. (1995)**, 'Review of (a) *Normal families of meromorphic functions*, by Chi-tai Chung, and (b) *Normal families*, by Joel L. Schiff'. *Bulletin of the American Mathematical Society (N.S.)*, volume **32**, pages 257–260.

[p. 258. First line of Theorem.]

Let D be a domain, $a, b \neq 0$ two complex numbers and $k \geq 1$ an integer.

113. Dubins, L. E. (1977), 'Group decision devices'. *American Mathematical Monthly*, volume **84**, pages 350–356.

[p. 353. Line 6.]

Proof. For each d, $u(d, \cdot)$ has an inverse function $u^{-1}(d, \cdot)$, where

$$u(d, c) = t \leftrightarrow u^{-1}(d, t) = c.$$

114. (25, 126, 177, 179, 252, 268) **Duke, W. (1997)**, 'Some old problems and new results about quadratic forms'. *Notices of the American Mathematical Society*, volume **44**, pages 190–196.

[p. 193. Lines 9–13, second column.]

...it can be seen that the number of representations of n as a sum of four squares is eight times the sum of those divisors of n which are not multiples of four. In particular, it is never zero!

115. (120, 232) **Dummigan, N. (1995)**, ‘The determinants of certain Mordell-Weil lattices’. *American Journal of Mathematics*, volume **117**, pages 1409–1429.

[p. 1419. Last line of Definition 1..]

We denote the circle diagram by enclosing the string diagram in square brackets, e.g. $[XOXOX]$.

116. (14) **Duren, P., W. Hengartner, and R. S. Laugesen (1996)**, 'The argument principle for harmonic functions'. *American Mathematical Monthly*, volume **103**, pages 411–415.

[p. 413. Lines 13–14.]

However, the singular zeroes of a harmonic function are not always isolated.

117. (56) **Dym, C. L. and E. S. Ivey (1980)**, *Principles of Mathematical Modeling.* Academic Press.

[p. 76. Lines 5–7.]

(The assumption that such discrete data can be plotted as a continuous curve is often referred to as the *continuum hypothesis*.)

118. (33, 117) **Edelman, A. and E. Kostlan (1995)**, 'How many zeros of a random polynomial are real?'. *Bulletin of the American Mathematical Society (N.S.)*, volume **32**, pages 1–37.

[p. 7. Lines 6–5 from the bottom.]

If $v(t)$ is the moment curve, then we may calculate $\|\gamma'(t)\|$ with the help of the following observations and some messy algebra:

119. (29, 133, 229, 261) **Edgar, G. A., D. H. Ullman, and D. B. West, editors (1997)**, 'Problems and solutions'. *American Mathematical Monthly*, volume **104**, pages 566–576.

[p. 574. Problem 10426.]

Show that any integer can be expressed as a sum of two squares and a cube. Note that the integer being represented and the cube are both allowed to be negative.

120. (133) **Ömer Eğecioğlu (1992)**, 'A combinatorial generalization of a Putnam problem'. *American Mathematical Monthly*, volume **99**, pages 256–258.

[p. 257. Lines 14–16.]

Setting

$$f(t) = c_{q-1}\xi^{q-1} + \cdots + c_1\xi + c_0,$$

we then have $f(\xi) = 0$. Furthermore, $f(t)$ has integral coefficients.

121. (155) **Einstein, A. and V. Bargmann (1944)**, 'Bivector fields'. *The Annals of Mathematics, 2nd Ser.*, volume **45**, pages 1–14.

[p. 1. Lines 8–17.]

The theory of gravitation is determined by the following formal elements:

1. The four-dimensional space-time continuum.
2. The covariance of the field equations with respect to all continuous coordinate transformations.
3. The existence of a Riemannian metric (i.e. a symmetric tensor g_{ik} of the second rank) which defines the structure of the physical continuum.

Under these circumstances the gravitational equations are the simplest consitions for the functions g_{ik} which restrict them to a sufficient degree.

We have tried to retain 1) and 2) but to describe the space structure by a mathematical object different from 3) yet resembling in some way the g_{ik}.

122. (78, 125, 127, 140, 204) **Eklof, P. C. (1976)**, 'Whitehead's problem is undecidable'. *American Mathematical Monthly*, volume **83**, pages 775–788.

[p. 786. Lines 7–6 from bottom.]

7.5 Lemma. For any uncountable subset P' of P there is a free subgroup A' which is pure in A and an uncountable subset P'' of P' such that $\mathrm{dom}(\phi) \subseteq A'$ for every ϕ in P'.

123. (33, 210) **Epp, S. S. (1995)**, *Discrete Mathematics with Applications, 2nd Ed.* Brooks/Cole. [p. 2. Blue box.]

124. (68, 73, 171, 175, 177) **Epp, S. S. (1995)**, *Discrete Mathematics with Applications, 2nd Ed.* Brooks/Cole. [p. 76. Lines 12–11 from bottom.]

125. (81) **Epp, S. S. (1995)**, *Discrete Mathematics with Applications, 2nd Ed.* Brooks/Cole. [p. 260. Line 3.]

126. (69, 79, 115, 123, 202, 217) **Epp, S. S. (1995)**, *Discrete Mathematics with Applications, 2nd Ed.* Brooks/Cole. [p. 534. Blue box.]

127. (173) **Erdös, P. and P. Turán (1937)**, 'On interpolation I'. *The Annals of Mathematics, 2nd Ser.*, volume **38**, pages 142–155.

[p. 147. Lines 8–11.]

If $\alpha = \beta = 0$, we have the Legendre-polynomials $P_n(x)$; for $\alpha = \beta = \frac{1}{2}$ the Tschebischeff polynomials $T_n(x)$.

128. (87, 142, 181, 258) **Exner, G. R. (1996)**, *An Accompaniment to Higher Mathematics*. Springer-Verlag.

[p. 35. Theorem 1.115.]

Suppose E is an equivalence relation on a set S. For any x in S, denote by E_x the set of all y in S equivalent under E to x. Then the collection of all E_x is a partition of S.

129. (99, 117, 120) **Farrell, F. T. and L. E. Jones (1989)**, 'A topological analogue of Mostow's rigidity theorem'. *Journal of the American Mathematical Society*, volume , pages 257–370.

[p. 272. Lemma 2.1.]

The following inequalities hold for any vector η *tangent to a leaf of the foliation* $\mathcal{F}$ *of* FM*.*

$$2h\,(dq(\eta), dq(\eta)) \geq \hat{h}(\eta, \eta) \geq h\,(dq(\eta), dq(\eta))$$

where $q : \mathcal{D}M \to M$ *denotes the bundle projections.*

130. (131, 138, 253) **Fearnley-Sander, D. (1982)**, 'Hermann Grassmann and the prehistory of universal algebra'. *American Mathematical Monthly*, volume **89**, pages 161–166.

[p. 161. Lines 16–12 from bottom.]

Given a set of symbols $S = \{x_1, x_2, \ldots, x_n\}$, the set

$$G = \{x_1, x_2, \ldots, x_n, (x_1x_1), (x_1x_2), \ldots, \\ (x_nx_n), x(x_1(x_1x_1)), (x_1(x_1x_2)), \ldots, \\ (x_1(x_nx_n)), (x_2(x_1x_1)), \ldots, \\ (x_n(x_nx_n)), ((x_1x_1)x_1), ((x_1x_1)x_2), \ldots\}$$

obtained by repeated juxtaposition of the symbols already written down, forms a groupoid in a natural way (the binary operation being juxtaposition).

131. (232) **Feeman, T. G. and O. Marrero (2000)**, 'Sequences of chords and of parabolic segments enclosing proportional areas'. *The College Mathematics Journal*, volume **31**, pages 379–382.

[p. 381. Second to last displayed formula.]

$$1 \leq \frac{L_n^2}{d_n^2} \leq \frac{\left[1 + c\left(a_{n+1} + a_n\right)\right]^2}{1 + c^2\left(a_{n+1} + a_n\right)^2} \to \frac{c^2}{c^2} = 1$$

132. (118, 209) **Finn, R. (1999)**, 'Capillary surface interfaces'. *Notices of the American Mathematical Society*, volume **46**, pages 770–781.

[p. 774. Formula (11) and the line above.]

The same procedure with $\Omega^* = \Omega$ yields

$$2H = \frac{|\Sigma| \cos\gamma}{|\Omega|}$$

133. (51, 65, 78, 79, 266) **Fisher, D. (1982)**, 'Extending functions to infinitesimals of finite order'. *American Mathematical Monthly*, volume **89**, pages 443–449.

[p. 445. Lines 9–11.]

(i) if f is in the domain of T, and $\mathrm{dom}(f)$ is the domain of f, then the function with domain $\mathrm{dom}(f)$ and constant value 1 is in the domain of T ...

134. (123) **M. K. Fort, J. (1052)**, ‘Differentials’. *American Mathematical Monthly*, volume **59**, pages 392–395.

[p. 394. Line 15.]

We define the identity function I by letting $I(x) = x$ for each real number x.

135. (98, 160) **L. R. Ford, J. (1957)**, 'Solution of a ranking problem from binary comparisons'. *American Mathematical Monthly*, volume **64**, pages 28–33.

[p. 28. Last three lines).]

We shall actually solve the following equivalent problem: In the set $\{w_i > 0; \Sigma w_i = 1\}$, find values w_i that maximize

$$F_A(w) = \Pi_{i<j} \left(\frac{w_i}{w_i + w_j} \right)^{a_{ij}} \left(\frac{w_j}{w_i + w_j} \right)^{a_{ji}}$$

136. (173) **Forsythe, G. E. (1957)**, 'Generation and use of orthogonal polynomials for data-fitting with a digital computer'. *Journal of the Society for Industrial and Applied Mathematics*, volume **5**, pages 74–88.

[p. 79. Lines 11–9 from bottom.]

In a data fitting code [8] written at the Lockheed Aircraft Company, for example, $p_i(x)$ was selected to be the ith Chebyshov polynomial $T_i(x)$ over an interval containing all the x_μ.

137. (214) **Fournelle, T. A. (1993)**, 'Symmetries of the cube and outer automorphisms of S_6'. *American Mathematical Monthly*, volume **100**, pages 377–380.

[p. 377. Line 9 above picture.]

To begin, recall that an *isometry* of $\mathbb{R}^3$ is a bijection which preserves distance.

138. (117, 133, 134, 135, 259) **Fowler, D. (1996)**, 'The binomial coefficient function'. *American Mathematical Monthly*, volume **103**, pages 1–17.

[p. 2. Lines 1–4 of Section 2.]

The factorial function can be defined by $x! = \Gamma(x+1) = \int_0^\infty t^x e^{-t} dt$ for $x > -1$, extended uniquely to negative nonintegral x by defining $x! = (x+n)!/(x+n)(x+n-1)\ldots(x+1)$ for any integer n such that $(x+n) > -1$.

139. (69, 201, 228, 258) **Fraleigh, J. B. (1982)**, *A First Course in Abstract Algebra*. Addison-Wes-ley.

[p. 41. Lines 6–7.]

Finally, we remark that for $\phi : A \to B$, the set A is the **domain of** ϕ, the set B is the **codomain of** ϕ, and the set $A\phi = \{a\phi \mid a \in A\}$ is the **image of** A **under** ϕ.

140. (107, 193, 239) **Frantz, M. (1998)**, 'Two functions whose powers make fractals'. *American Mathematical Monthly*, volume **105**, pages 609–617.

[p. 609. Lines 4 and 5.]

...Richard Darst and Gerald Taylor investigated the differentiability of functions f^p (which for our purposes we will restrict to $(0,1)$) defined for each $p \geq 1$ by

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1/n^p & \text{if } x = m/n \text{ with } (m,n) = 1. \end{cases}$$

141. (222) **Frantz, M. (1998)**, 'Two functions whose powers make fractals'. *American Mathematical Monthly*, volume **105**, pages 609–617.

[p. 614. Formula (4).]

...if $p > 1$ and B is the set of numbers x that are not dyadic rationals and satisfy

$$|x - m/2^n| \leq (2^n)^{-p}$$

for infinitely many dyadic rationals $m/2^n$, then...

142. (34, 52, 79) **Freiling, C. (1990)**, 'Symmetric derivates, scattered, and semi-scattered sets'. *Transactions of the American Mathematical Society*, volume **318**, pages 705–720.

[p. 705. Abstract.]

We call a set right scattered (left scattered) if every non-empty subset contains a point isolated on the right (left).

143. (75, 142, 182) **Freiling, C. (1990)**, 'Symmetric derivates, scattered, and semi-scattered sets'. *Transactions of the American Mathematical Society*, volume **318**, pages 705–720.

[p. 715. Lines 15–14 and 7–6 from bottom.]

Theorem 3. If F is an interval function satisfying condition (A) on an open interval I such that ...

Proof. Let F be as stated and let A, B and C be disjoint sets where $A \cup B \cup C = I \ldots$

144. (59, 137) **Friedman, N. A. (1992)**, 'Replication and stacking in ergodic theory'. *American Mathematical Monthly*, volume **99**, pages 31–41.

[p. 33. Lines 13–10 from bottom.]

In general, let $(X_i, \mathcal{B}_i, m_i)$ be measure spaces with $m_i(X_i) = 1$, $i = 1, 2$. Let ϕ be an invertible mapping from X_1 to X_2 such that $m_1(B_1) = m_2(\phi(B_1))$, $B_1 \in \mathcal{B}_1$, and $m_2(B_2) = m_1(\phi^{-1}(B_2))$, $B_2 \in \mathcal{B}_2$. We refer to ϕ as an *isomorphism*. Transformations T_i on X_i, $i = 1, 2$ are *isomorphic* if there exists an isomorphism ϕ such that $T_1(x) = \phi^{-1}(T_2(\phi(x)))$ for $x \in X_1$. We refer to T_2 as a *copy* of T_1.

145. (35, 166) **Friedman, W. F. and C. J. Mendelsohn (1932)**, 'Notes on code words'. *American Mathematical Monthly*, volume **39**, pages 394–409.

[p. 407. Lines 10–7 from bottom.]

We may, however, add an extra character to the alphabet, giving $(\lambda + 1)$ characters, construct a table without transpositions, and then eliminate all words containing the extra character.

146. (214, 265) **Friedman, R. (1995)**, 'Vector bundles and $SO(3)$-invariants for elliptic surfaces'. *Journal of the American Mathematical Society*, volume **8**, pages 29–139.

[p. 29. Lines 4–6.]

Recall that a relatively minimal simply connected elliptic surface S is specified up to deformation type by its geometric genus $p_g(S)$ and by two relatively prime integers m_1, m_2, the multiplicities of its multiple fibers.

147. (181, 204, 250) **Furi, M. and M. Martelli (1991)**, 'The teaching of mathematics'. *American Mathematical Monthly*, volume **98**, pages 835–846.

[p. 842. Lines 17–20.]

By applying the Mean Value Theorem to f on $[a, d]$ and $[d, b]$ respectively, we obtain

$$f(d) - f(a) = f'(c_1)(d - a), \qquad c_1 \in (a, d),$$
$$f(b) - f(d) = f'(c_2)(b - d), \qquad c_1 \in (d, b).$$

148. (50) **Fulda, J. S. (1989)**, 'Material implication revisited'. *American Mathematical Monthly*, volume **96**, pages 247–250.

[p. 247. Lines 1–3.]

Material implication, the logical connective corresponding to the English "if" . . .

149. (9) **Fulda, J. S. (1989)**, 'Material implication revisited'. *American Mathematical Monthly*, volume **96**, pages 247–250.

[p. 248. Third paragraph.]

It is the thesis of this paper that this uneasiness is none other than the familiar temptation to commit the fallacy of conversion ($p \Rightarrow q \ \vdash\ q \Rightarrow p$), also known as the fallacy of affirming the consequence ($p \Rightarrow q, q \ \vdash\ p$) ...

150. (25, 64, 245) **Galvin, F. (1994)**, 'A proof of Dilworth's chain decomposition theorem'. *American Mathematical Monthly*, volume **101**, pages 352–353.

[p. 352. Lines 1–2.]

Let P be a finite partially ordered set: a *chain (antichain)* in P is a set of pairwise comparable (incomparable) elements; the *width* of P is the maximum cardinality of an antichain in P.

151. (173) **Geddes, K. O. (1981)**, 'Block structure in the Chebyshev-Padé table'. *SIAM Journal on Numerical Analysis*, volume **18**, pages 844–861.

[p. 844. Title.]

Block Structure in the Chebyshev-Padé Table

152. (115, 239) **Gelbaum, B. R. and J. M. H. Olmsted (1990)**, *Theorems and Counterexamples in Mathematics.* Springer-Verlag.

[p. 65. Exercise 2.1.2.14.]

Show that for f in **Exercise 2.1.1.13. 49**, if g is given by

$$g(x) \stackrel{\text{def}}{=} \int_0^x f(t)dt$$

then: ...

153. (148, 181) **Gelbaum, B. R. and J. M. H. Olmsted (1990)**, *Theorems and Counterexamples in Mathematics.* Springer-Verlag.

[p. 85. Theorem 2.1.3.5..]

Let (2.1.3.3) obtain everywhere on a measurable set E of positive measure. Then ...

154. (32, 126) **Geoffrion, A. M. (1071)**, 'Duality in nonlinear programming: A simplified applications-oriented development'. *SIAM Review*, volume **13**, pages 1–37.

[p. 19. Lines 3–5.]

In other words, $y = 0$ is always a boundary point of Y. This turns out to be true in general when (P) has a finite optimal value but is unstable, as follows from the next theorem.

155. (14, 141, 182, 204) **Giblin, P. J. and S. A. Brassett (1985)**, 'Local symmetry of plane curves'. *American Mathematical Monthly*, volume **92**, pages 689–707.

[p. 691. Line 6 under the figure.]

Suppose we are given a smooth simple closed plane curve and a point p on it. Is there always a circle or straight line tangent to the curve at p and at another point $p' \neq p$?

156. (47, 67, 69, 128) **Giesy, D. P. (1971)**, 'The general solution of a differential-functional equation'. *American Mathematical Monthly*, volume **78**, pages 37–42.

[p. 37. Lines 10–1.]

2Definition. A function g is *of type* (A) *on* I if g has an antiderivative v on I and

$$g(a) = g(b) \textit{ implies } v(a) = v(b) \textit{ for all } a \textit{ and } b \textit{ in } I$$

157. (98, 117) **Gilmore, P. C. (1960)**, 'An alternative to set theory'. *American Mathematical Monthly*, volume **67**, pages 621–632.

[p. 622. Lines 5–3 from bottom.]

The nu relation between symbols is such that the following sentences are true:

$$2\ \nu \text{ odd}, \quad 4\ \nu \text{ odd}, \quad 1\ \nu \text{ even}, \quad 3\ \nu \text{ even}.$$

158. (173) **Gordon, B. (1958)**, ‘On a Tauberian theorem of Landau’. *Proceedings of the American Mathematical Society*, volume **9**, pages 693–696.

[p. 693. Line 1.]

It was the plan ot Tschebyshef [1] and Sylvester [2] to deduce the prime number theorem . . .

159. (120, 249) **Gottlob, G. (1997)**, 'Relativized logspace and generalized quantifiers over finite ordered structures'. *Journal of Symbolic Logic*, volume **62**, pages 545–574.

[p. 571. Lines 6–7 under (9).]

Projection translations are first order translations whose translation formula is *projective*, i.e., of a special, particularly simple form.

160. (101, 276) **Gould, H. W. (1972)**, 'Explicit formulas for Bernoulli numbers'. *American Mathematical Monthly*, volume **79**, pages 44–51.

[p. 44. Lines 12–13.]

Basically, what Higgins found when $a = 0$ in his general formula (2.5) is

$$B_n = \sum_{k=0}^{n} \frac{1}{k+1} \sum_{j=0}^{k} (-1)^j \binom{k}{j} j^n, \quad n \geq 0,$$

161. (101, 194) **Gouvea, F. Q. (1994)**, ' ""A marvelous proof'. *American Mathematical Monthly*, volume **101**, pages 203–222.

[p. 213. Line 7.]

Let $\mathfrak{h} = \{x + iy \mid y > 0\}$ be the complex upper half-plane.

162. (51, 238) **Gowers, W. T. (2000)**, 'The two cultures of mathematics'. In [Arnold *et al.*, 2000], pages 65–78.

[p. 74. Lines 15–12.]

THEOREM. *Let* G *be a graph, let* $\epsilon > 0$ *and let* k *be a positive integer. Then there exists a constant* K*, depending on* k *and* ϵ *only, such that the vertices of* G *can be partitioned into* m *sets* $A_1, \ldots, A_m$*, with* $k \leq m \leq K$*, such that at least* $(1-\epsilon)\binom{m}{2}$ *of the pairs* (A_i, A_j) *(with* $i < j$*) are* ϵ*-uniform.*

163. (47) **Graham, N., R. C. Entringer, and L. A. Szekely (1994)**, 'New tricks for old trees: Maps and the pigeonhole principle'. *American Mathematical Monthly*, volume **101**, pages 664–667.

[p. 665. Lines 4–3 from bottom.]

Assume ϕ has no fixed vertex. Then, for every vertex v, there is a unique non-trivial path in T from v to $\phi(v)$.

164. (56, 105, 266, 267) **Graham, R. L., D. E. Knuth, and O. Patashnik (1989)**, *Concrete Mathematics.* Addison-Wes-ley.

[p. 71. Lines 15–13 from bottom.]

Let $f(x)$ be a continuous, monotonically increasing function with the property that

$$f(x) = \text{integer} \Rightarrow x = \text{integer}$$

(The symbol '$\Rightarrow$' means "implies.")

165. (275, 276) **Graham, C. C., A. T. Lau, and M. Leinert (1991)**, 'Continuity of translation in the dual of $L^\infty(G)$ and related spaces'. *Transactions of the American Mathematical Society*, volume **328**, pages 589–618.

[p. 589. Line 5 from bottom.]

When $X = X_c$, $\mu * f$ is well-defined for every $\mu \in M(G)$ and every $f \in X$.

166. (32) **Grayson, M., C. Pugh, and M. Shub (1994)**, 'Stably ergodic diffeomorphisms'. *Annals of Mathematics, 2nd Ser.*, volume **140**, pages 295–329.

[p. 304. Lines 7–6 from bottom.]

This is the usual picture of the Lie bracket $[X, Y] = Z$, and becomes especially clear if drawn in a flowbox for X.

167. Grassman, W. K. and J.-P. Tremblay (1996), *Logic and Discrete Mathematics: A Computer Science Perspective*. Prentice-Hall.

[p. 234. Definition 5.3.]

A is a *proper subset* of B if A is a subset of B but A is not equal to B. If A is a proper subset of B, we write $A \subset B$.

168. (39) **Grimaldi, R. P. (1999)**, *Discrete and Combinatorial Mathematics, An Applied Introduction, Fourth Edition.* Addison-Wes-ley.

[p. 128. Example 3.2(a).]

$A = \{1, 4, 9, \ldots, 64, 81\} = \{x^2 \mid x \in \mathcal{U}, x^2 < 100\} = \{x^2 \mid x \in \mathcal{U} \wedge x^2 < 100\} = \{x^2 \in \mathcal{U} \mid x^2 < 100\}$.

169. (148) **Greenlaw, R. and H. J. Hoover (1998)**, *Fundamentals of the Theory of Computation.* Morgan Kaufmann Publishers, Inc.

[p. 36. Footnote.]

All logarithms in this text are base two unless noted otherwise. We use the now quite common notation $\lg x$ instead of $\log_2 x$.

170. (65, 78, 147) **Griewank, A. and P. J. Rabier (1990)**, 'On the smoothness of convex envelopes'. *Transactions of the American Mathematical Society*, volume **322**, pages 691–709.

[p. 692. Lines 15–18.]

... because the construction of conv E involves global properties of E, appropriate smoothness of the boundary of $\mathrm{dom}(E)$, the domain of E, may have to be required near the points of some subset of $\partial\,(\mathrm{dom}(E)) \cap \mathrm{dom}(E)$.

171. (17) **Gries, D. and F. B. Schneider (1995)**, ‘Teaching math more effectively, through calculational proofs’. *American Mathematical Monthly*, volume **102**, pages 691–697.

[p. 693. Lines 10–11.]

Notice that the proof format makes it easy to find the facts on which the proof depends –they are given within the angle brackets $\langle$ and $\rangle$.

172. (14, 105, 182, 274) **Grove, V. G. (1934)**, 'On a certain correspondence between surfaces in hyperspace'. *Transactions of the American Mathematical Society*, volume **36**, pages 627–636.

[p. 627. Lines 1–5.]

Consider a surface S and a point x on S. Let the parametric vector equation of S be

$$x = x(u, v).$$

The ambient space of the osculating planes at the point x to all of the curves through x is a certain space $S(2, 0)$ called *the two-osculating space of S at x.*

173. (160, 209) **Gross, K. L. (1978)**, 'On the evolution of noncommutative harmonic analysis'. *American Mathematical Monthly*, volume **85**, pages 525–548.

[p. 537. Lines 16–17.]

1. A connected compact *abelian* Lie group is necessarily a *torus*, by which is meant a direct product of circles.

174. (77, 106) **Groetsch, C. W. (1993)**, 'Inverse problems and Torricelli's law'. *The College Mathematics Journal*, volume **24**, pages 210–217.

[p. 210. Lines 5–8.]

Because K is a function, a unique output v exists for each input u in the domain of K and if K is in some sense *continuous*, as is very often the case, the output v depends continuously on u.

175. (52) **Guckenheimer, J. and S. Johnson (1990)**, 'Distortion of S-unimodal maps'. *Annals of Mathematics, 2nd Ser.*, volume **132**, pages 71–130.

[p. 72. Lines 16–18.]

Our third result examines maps that have "sensitive dependence to initial conditions". These are maps whose non-wandering set contains an interval.

176. (73, 143) **Gulick, S. L. (1968)**, 'The minimal boundary of $C(X)$'. *Transactions of the American Mathematical Society*, volume **131**, pages 303–314.

[p. 308. Lines 1–3.]

Let $Y = [0, 1]$ with the usual topology, and let Z be a non-Borel subset of Y, which by the axiom of choice exists. Let the elements of Z be denoted by $\{y_\lambda : \lambda \in \Lambda\}$, Λ an index set.

177. (189, 212) **Gulliver II, R. D., R. Osserman, and H. L. Royden (1973)**, 'A theory of branched immersions of surfaces'. *American Journal of Mathematics*, volume **95**, pages 750–812.

[p. 757. Lines 7–10.]

We may think of them as

$$x_k = \bar{\phi}_k\,(x_1, x_2) = \phi_k\left((x_1 + ix_2)^{\frac{1}{m}}\right), \qquad k = 3, \ldots, n, \tag{1.11}$$

where $\bar{\phi}_k$ is a multi-valued function of x_1, x_2, defining an m-sheeted surface over a neighborhood of the origin.

178. (8, 21, 37, 125, 147, 250) **Hadwin, D. (1994)**, 'A general view of reflexivity'. *Transactions of the American Mathematical Society*, volume **344**, pages 325–360.

[p. 328. Lines 11–6 from bottom.]

In this section we assume that the field F is either the real or complex numbers. ... Throughout this section let $\tilde{E} = \{e \in E : \|e\| = 1\}$, and for any subset B of Y, let $\overline{co}(B)$ denote the $\sigma(Y, X)$-closed convex hull of B in Y.

179. (128, 160) **Hailperin, T. (1981)**, 'Boole's algebra isn't Boolean Algebra'. *Mathematics Magazine*, volume **54**, pages 172–184.

[p. 175. Lines 20–22.]

We need to distinguish **a Boolean algebra** from the *general concept of*, or the *formal theory of*, Boolean algebras. When using the indefinite article, we are referring to a particular mathematical structure which, via an appropriate interpretation, satisfies the above axioms.

180. (110, 204) **Haimo, F. and M. Tretkoff (1981)**, 'Enlarging a free subgroup of a symmetric group freely'. *Proceedings of the American Mathematical Society*, volume **82**, pages 31–35.

[p. 32. Lines 17–18.]

Let ψ'_h be ψ_h with its codomain extended from S_h to Sym$M$ ...

181. (50, 52, 161) **Halmos, P. R. (1956)**, 'The basic concepts of algebraic logic'. *American Mathematical Monthly*, volume **63**, pages 363–367.

[p. 365. Last two lines.]

Suppose, moreover, that A and N are two distinct objects not contained in S; intuitively A and N are to be thought of as the connectives "and" and "not".

182. (9, 56) **Halmos, P. R. (1990)**, 'Has progress in mathematics slowed down?'. *American Mathematical Monthly*, volume **97**, pages 561–568.

[p. 580. Lines 15–20.]

The simplest statement of Cantor's continuum hypothesis ... is that every uncountable set of real numbers is in one-to-one correspondence with the set of all real numbers, or, in Cantor's notation, that there is no cardinal number between $\aleph_0$ and $2^{\aleph_0}$.

183. (9, 101) **Halbeisen, L. and S. Shelah (2001)**, 'Relations between some cardinals in the absence of the axiom of choice'. *The Bulletin of Symbolic Logic*, volume **7**, pages 237–251.

[p. 239. Lines 15–16.]

We will use fraktur-letters to denote cardinals and $\aleph$'s to denote the alephs.

184. (117, 118) **Harris, M. (1993)**, 'L-functions of 2×2 unitary groups and factorization of periods of Hilbert modular forms'. *Journal of the American Mathematical Society*, volume **6**, pages 637–719.

[p. 639. Lines 9–8 from bottom.]

Following the pattern first observed by Waldspurger, the vanishing of $\Theta(\pi, \omega)$ to $GU_{\mathcal{H}}E(D)$ either vanishes or equals $\left(\breve{\pi}^D, \omega^{-1}\right)$, ...

185. (121) **Hardman, N. R. and J. H. Jordan (1967)**, 'A minimum problem connected with complete residue systems in the Gaussian integers'. *American Mathematical Monthly*, volume **74**, pages 559–561.

[p. 559. Lines 1–2.]

A Gaussian integer, γ, is a complex number that can be expressed as $\gamma = a + bi$, where a and b are real integers and i is the so-called imaginary unit.

186. (122, 137, 245) **Hassett, M. J. (1969)**, 'Recursive equivalence types and groups'. *Journal of Symbolic Logic*, volume **34**, pages 13–20.

[p. 13. Lines 8–10.]

Then we can identify the group $\mathcal{B}_\sigma$ with the group $P(\sigma)$ consisting of the set $\sigma! = \{f^* \mid f \in \mathcal{B}_\sigma\}$ under the multiplication "·" defined by $f^* \cdot g^* = (fg)^*$.

187. (51) **Hastings, S. P. and J. B. McLeod (1993)**, 'Chaotic motion of a pendulum with oscillatory forcing'. *American Mathematical Monthly*, volume **100**, pages 563–572.
[p. 564. Lines 5–2.]

Applying Newton's law of motion, we obtain the ordinary differential equation

$$ml\ddot{u} + c\dot{u} + mg \sin u = 0,$$

where c is the positive constant of proportionality.

188. (78) **Hassell, C. and E. Rees (1993)**, 'The index of a constrained critical point'. *American Mathematical Monthly*, volume **100**, pages 772–778.

[p. 772. Lines 4–6.]

To find the critical points of a smooth function f defined on $M^n \subset \mathbb{R}^{n+m}$, a smooth submanifold given as the common zero-set of m smooth functions $g_i : \mathbb{R}^{n+m} \to \mathbb{R}$.

189. (93, 100, 232) **Hassell, C. and E. Rees (1993)**, 'The index of a constrained critical point'. *American Mathematical Monthly*, volume **100**, pages 772–778.

[p. 774. Lines 17–20.]

Fact 2. Let b be a non-degenerate symmetric bilinear form on V of dimension $2m$ and let $W \subset V$ have dimension m and $W \subset W^{\perp}$. Then b is represented by the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Proof: Choose $\underline{w} \neq 0$ in W and $\underline{v}$ such that $b(\underline{v}, \underline{w}) = 1$.

...

190. (20) **Hatcher, W. S. (1982)**, 'Calculus is algebra'. *American Mathematical Monthly*, volume **89**, pages 362–370.

[p. 363. Lines 10–12.]

In each case, the extension defined on R^* must be identical on R with the original function or relation defined on the field R, and of the same arity.

191. (175, 180) **Hathaway, A. S. (1887)**, 'A memoir in the theory of numbers'. *American Journal of Mathematics*, volume **9**, pages 162–179.

[p. 162. Lines 4–6.]

The labors of Gauss, Kummer, Dirichlet, Kronecker, Dedekind, and others, have extended the scope of the theory of numbers far beyond its original limit of the science of the natural numbers $0, \pm 1, \pm 2, \pm 3, \ldots$.

192. (185, 204) **Haws, L. and T. Kiser (1995)**, 'Exploring the brachistochrone problem'. *American Mathematical Monthly*, volume **102**, pages 328–336.

[p. 333. Lines 10–9 from bottom.]

... to obtain the 2nd order differential equation

$$\left(1 + (y')^2\right)(1 + \mu y') + 2\,(y - \mu x)\,y'' = 0.$$

193. (122) **Hadwin, D. and J. W. Kerr (1988)**, 'Scalar-reflexive rings'. *Proceedings of the American Mathematical Society*, volume **103**, pages 1–8.

[p. 2. Lines 9–8.]

Suppose R is a commutative ring with identity and the R-module M is the direct sum of a family $\{Rx_k : k \in K\}$ of cyclic submodules.

194. (20, 258) **Hébert, M., R. N. Mckenzie, and G. E. Weaver (1989)**, 'Two definability results in the equational context'. *Proceedings of the American Mathematical Society*, volume **107**, pages 47–53.

[p. 47. Lines 3–4.]

Definitions 1. (i) A *type* τ is a set of operation symbols, each with an assigned *arity* (some cardinal). τ is *bounded* by the infinite regular cardinal α if the arities of its operation symbols are all $< \alpha$.

195. (47, 48, 93, 127) **Henriksen, M., S. Larson, J. Martinez, and R. G. Woods (1994)**, 'Lattice-ordered algebras that are subdirect products of valuation domains'. *Transactions of the American Mathematical Society*, volume **345**, pages 195–221.

[p. 213. Lines 6–10 under Proposition 5.1.]

... $QF(X)$ is an F-space if and only if:

(*) If C_1, C_2 are disjoint cozero sers, then there are zero sets Z_1, Z_2 such that $C_1 \subseteq Z_1$, $C_2 \subseteq Z_2$ and $\operatorname{int}(Z_1 \cap Z_2) = \emptyset$.

Thus (*) is a sufficient but not necessary condition for $QF(X)$ to be an SV-space.

196. (76, 79, 249) **Herstein, I. N. (1964)**, *Topics in Algebra.* Blaisdell.

[p. 2. Lines 11–14.]

The set A will be said to be a *subset* of the set S if every element in A is an element of S, that is, if $a \in A$ implies that $a \in S$. We shall write this as $A \subset S$ ... This notation is not meant to preclude the possibility that $A = S$.

197. (173) **Higgins, T. J. (1944)**, 'Biographies and collected works of mathematicians'. *American Mathematical Monthly*, volume **51**, pages 433–445.

[p. 440. Lines 10–7 from bottom.]

Portraits of Eminent Mathematicians with Brief Biographical Sketches. Portfolio No. 2. By D. E. Smith. New York, Scripta Mathematica, 1938. Euclid, Cardan, Kepler, Fermat, Pascal, Euler, Laplace, Cauchy,Jacobi, Hamiltohn, Cayley, Chebishef, Poincaré.

198. (268) **Hills, E. J. (1948)**, 'Fundamentals of beginning algebra'. *Mathematics Magazine*, volume **21**, pages 212–230.

[p. 213. Lines 16–12 from bottom.]

Addition of products

English statements	Arithmetic statements
Two times three plus three times four	$2 \times 3 + 3 \times 4$
Two times four plus three times six	$2 \times 4 + 3 \times 6$

199. (170, 177) **Hille, E. (1948)**, 'Non-oscillation theorems'. *Transactions of the American Mathematical Society*, volume **64**, pages 234–252.

[p. 241. Lines 12–9 from bottom.]

It follows that $y'(x)$ is never increasing for $a < x$ and $y(x)$ is concave downwards. Since the graph of $y = y(x)$ lies below the curve tangent and does not intersect the x-axis for $x > a$, we must have $y'(x) > 0$.

200. (56, 150, 178) **Hocking, J. G. and G. S. Young (1961)**, *Topology.* Addison-Wes-ley.

[p. 13. .]

We shall refer to a continuous transformation as a *mapping* from now on.

201. (236) **Hochstrasser, U. (1962)**, 'Numerical methods for finding solutions of nonlinear equations'. In *Survey of Numerical Analysis*, Todd, J., editor. McGraw-Hill.

[p. 264. Lines 11–15.]

By repeated application of this squaring method, polynomials ${}_kP_n(z) = \sum_{j=0}^{n} {}_ka_{n-j}z^j$ can be obtained, the roots of which are equal to $-x_i^{2^k}$, and the coefficients are determined reecursively by

$$ {}_ka_j = {}_{k-1}a_j^2 + 2\sum_{i=1}^{j}(-1)^l {}_{k-1}a_{j+1}\, {}_{k-1}a_{j-1} $$

202. (219, 265) **Hodgson, J. P. E. (1984)**, 'Surgery on Poincaré complexes'. *Transactions of the American Mathematical Society*, volume **285**, pages 685–701.

[p. 685. Lines 9–4.]

This paper has the following organization. §1 contains the definitions and notations that will be used. §2 contains some technical results on Poincaré complexes and the central lemma which makes everything work and which we call the Surgery Preparation Lemma. The remaining paragraphs unwind the details, so that §3 contains the proof of the $(\pi - \pi)$ theorem for Poincaré duality spaces of formal dimension $\geq 7 \ldots$

203. (47, 250) **Hofmann, K. H. and C. Terp (1994)**, ‘Compact subgroups of Lie groups and locally compact groups’. *Proceedings of the American Mathematical Society*, volume **120**, pages 623–634.

[p. 630. Lines 19–18 from bottom.]

… and thus G_0/K_0 is homeomorphic to a euclidean space only if $C = K_0$.

204. (33, 84) **Holland, Jr., S. S. (1995)**, 'Orthomodularity in infinite dimensions; a theorem of M. Solèr'. *Bulletin of the American Mathematical Society (N.S.)*, volume **32**, pages 205–234.

[p. 206. Lines 5–4 from the bottom.]

Thus $(M + M^{\perp})^{\perp\perp} = 0^{\perp} = E$. But $M + M^{\perp}$ is closed, so $M + M^{\perp} = E$.

205. (47, 61, 78, 125, 147, 238, 251, 269) **Hofmann, S. and J. L. Lewis (1996)**, 'l^2 solvability and representation by caloric layer potentials in time-varying domains'. *The Annals of Mathematics, 2nd Ser.*, volume **144**, pages 349–420.

[p. 349. Lines 10–5.]

R. Hunt proposed the problem of finding an analogue of Dahlberg's result for the heat equation in domains whose boundaries are given locally as graphs of functions $A(x,t)$ which are Lipschitz in the space variable. It was conjectured at one time that A should be $\mathrm{Lip}_{\frac{1}{2}}$ in the time variable, but subsequent counterexamples of Kaufmann and Wu [KW] showed that this condition does not suffice.

206. (36, 60) **Holland, Jr., S. S. (1995)**, 'Orthomodularity in infinite dimensions; a theorem of M. Solèr'. *Bulletin of the American Mathematical Society (N.S.)*, volume **32**, pages 205–234.

[p. 222. line 10 from bottom.]

The set of those elements of L which are the join of finitely many atoms is closed under the operations $\vee$ and $\wedge$ ...

207. (41) **Howe, R. E. and E.-C. Tan (1993)**, 'Homogeneous functions on light cones: the infinitesimal structure of some degenerate principal series representations'. *Bulletin of the American Mathematical Society (N.S.)*, volume **28**, pages 1–74.

[p. 8. Lines 7–9.]

Thus our computation of the action of $\mathfrak{p}$ on individual K-types is in principle (and will turn out to be in practice) sufficient for understanding the submodule structure of $S^a(X^0)$.

208. (57) **Howe, E. W. (1995)**, 'Principally polarized ordinary Abelian varieties over finite fields'. *Transactions of the American Mathematical Society*, volume **347**, pages 2361–2401.

[p. 2385. First line of proof of Lemma 10.2.]

We will prove the contrapositive statement.

209. (212) **Hungerford, T. W. (1990)**, 'A counterexample in Galois theory'. *American Mathematical Monthly*, volume **97**, pages 54–57.

[p. 56. Lines 9–6 from bottom.]

But in modern algebra texts published since 1986 (and a few earlier ones), the reader should be more careful. Even when the Lemma is stated, its proof may be faulty. Most of these books present either a fallacious proof of the Theorem or Lemma (counterexample above) or a suspiciously incomplete proof.

210. (8) **Hunter, T. J. (1996)**, 'On the homology spectral sequence for topological Hochschild homology'. *Transactions of the American Mathematical Society*, volume **348**, pages 3941–3953.

[p. 3934. Lines 8–6 from bottom.]

We will often abuse notation by omitting mention of the natural isomorphisms making $\wedge$ associative and unital.

211. (69, 117, 136, 179, 239) **Ipsen, I. C. F. and C. D. Meyer (1995)**, 'The angle between complementary subspaces'. *American Mathematical Monthly*, volume **102**, pages 904–911.

[p. 905. First lines.]

Definition 2.1. For nonzero subspaces $\mathcal{R}, \mathcal{N} \subseteq \mathfrak{R}^n$, the minimal angle between $\mathcal{R}$ and $\mathcal{N}$ is defined to be the number $0 \leq \theta \leq \pi/2$ that satisfies

$$\cos\theta = \max_{\substack{\mathbf{u}\in\mathcal{R},\mathbf{v}\in\mathcal{N}\\ \|\mathbf{u}\|_2=\|\mathbf{v}\|_2=1}} \mathbf{v}^T\mathbf{u}$$

212. (266) **Jackson, D. (1934)**, 'The convergence of Fourier series'. *American Mathematical Monthly*, volume **41**, pages 67–84.

[p. 70. Lines 13–12 from bottom.]

It must be recognized however that the series has not yet been proved to converge *to the value* $f(x)$.

213. (125, 147, 197) **Jackson, B. W. and D. Thoro (1990)**, *Applied Combinatorics with Problem Solving.* Addison-Wes-ley.

[p. 55. Definition of permutation.]

Let X be a set with n different objects. An arrangement of all the elements of X in a sequence of length n is called a permutation.

214. (79) **Jackson, B. W. and D. Thoro (1990)**, *Applied Combinatorics with Problem Solving*. Addison-Wes-ley.

[p. 71. Problem 8.]

A small library contains 15 different books. If five different students simultaneously check out one book each, . . .

215. (126, 134, 264) **Jain, D. L. and R. P. Kanwal (1971)**, 'An integral equation method for solving mixed boundary value problems'. *SIAM Journal on Applied Mathematics*, volume **20**, pages 642–658.

[p. 643. Lines 7–2 from bottom.]

An integral equation of the form

$$\int_0^a K_0(t,\rho)g(t)\,dt = f(\rho), \qquad 0 < \rho < a,$$

with a given symmetric kernel K_0, embodies the solution of various mixed boundary value problems of physical interest. Being a Fredholm integral equation of the first kind, it cannot, in general, be solved easily for the unknown function $g(\rho)$ in terms of the given function $f(\rho)$.

216. (19, 106, 110) **James, R. C. (1951)**, 'Linear functionals as differentials of a norm'. *Mathematics Magazine*, volume **24**, pages 237–244.

[p. 237. Lines 10–8 from bottom.]

A continuous function F with real number values and argument in a Banach space B is a *real linear functional*, or simply *linear functional*, if $f(x + y) = f(x) + f(y)$ for each x and y of B.

217. (61, 61) **Jankovic, D. and T. R. Hamlet (1990)**, 'New topologies from old via ideals'. *American Mathematical Monthly*, volume **97**, pages 295–310.

[p. 300. lines 17–14 from bottom.]

By taking $\mathcal{I} = \mathcal{J}$ in the above theorem, the following corollary answers the question about the relationship between τ^* and τ^{**}.

218. (69, 168) **Jeffries, C. (1990)**, 'Code recognition with neural network dynamical systems'. *SIAM Review*, volume **32**, pages 636–651.

[p. 636. Lines 4–8.]

A mathematical neural network model is an n-dimensional difference equation or differential equation dynamical system that accounts for the dynamics of n *neurons*. Each neuron is mathematically a state x_i (a real number) with an associated output $g_i = g_i(x_i)$, typically a ramp or sigmoidal *gain function* $g_i(x_i)$.

219. (115, 133) **Jenkyns, T. and E. Muller (2000)**, 'Triangular triples from ceilings to floors'. *American Mathematical Monthly*, volume **107**, pages 634–639.

[p. 634. Line 1.]

A *triangular triple* is a sequence of non-negative integers (i, j, k) that gives the lengths of the sides of a triangle.

220. (21, 41) **Joag-Dev, K. and F. Proschan (1992)**, ‘Birthday problem with unlike probabilities’. *American Mathematical Monthly*, volume **99**, pages 10–12.

[p. 10. Lines 4–6.]

This computation is to be done under the assumption that individuals’ birthdays are independent and that for every individual, all 365 days of the year are equally likely as possible birthdays.

221. (202) **Jobe, W. H. (1962)**, ‘Functional completeness and canonical forms in many-valued logics’. *Journal of Symbolic Logic*, volume **27**, pages 409–422.

[p. 415. Lines 7–2.]

In order to avoid a prohibitive amount of punctuation, we assign the following order of precedence to the binary symbols $\equiv$, $\wedge$, $\cdot$. Of these, each has precedence over any listed to its right. To the unary symbols E_1 and E_2 we assign the *least precedence*. When two or more unary symbols occur in a sequence, the innermost symbol is given the least precedence.

222. (67, 168) **Jones, R. and J. Pearce (2000)**, 'A postmodern view of fractions and the reciprocals of Fermat primes'. *Mathematics Magazine*, volume **73**, pages 83–97.

[p. 95. Lines 16–17.]

DEFINITION. *A positive integer* $n > 1$ *is perfectly symmetric if its reciprocal is symmetric in any base* b *provided* $b \not\equiv 0 \pmod{n}$ *and* $b \not\equiv 1 \pmod{n}$.

223. (14, 168, 219) **Jurisic, A. (1996)**, 'The Mercedes knot problem'. *American Mathematical Monthly*, volume **103**, pages 756–770.

[p. 765. Lines 8–10 below Figure 15.]

Let i, j, k be the rotations of S_1 around the coordinate axes x, y, z, respectively, through π radians. Let us define a group generated by i, j, k modulo the equivalence relation of an ambient isotopy in a hollow ball H with the band A keeping S_1 and S_2 fixed.

224. (47, 99, 125) **Karlin, S. (1972)**, 'Some mathematical models of population genetics'. *American Mathematical Monthly*, volume **79**, pages 699–739.

[p. 706. Lines 12–16.]

Under these conditions adding the relations in (2.6) using obvious inequalities produces

$$x' + y' < 2\frac{xy + \frac{1}{2}(x+y)}{1 + \frac{1}{2}(x+y)} \tag{2.8}$$

Since $4xy \leq (x+y)^2$ we see that $x' + y' < x + y$. It follows that $x^{(n)} + y^{(n)}$ decreases in n and its limit is necessarily zero indicating that 0 is globally stable.

225. (60, 76) **Kalton, N. J. (1991)**, 'Differentials of complex interpolation processes for Köthe function spaces'. *Transactions of the American Mathematical Society*, volume **333**, pages 479–529.

[p. 479. Lines 11–13 of Introduction.]

The main idea here is to characterize those twisted sums of Köthe function spaces which can be obtained by differentiating a complex interpolation scale of Köthe function spaces.

226. (60, 76, 125) **Kang, M.-c. (1997)**, 'Minimal polynomials over cyclotomic fields'. *American Mathematical Monthly*, volume **104**, pages 260.

[p. 260. Lines 23–24.]

From the definition of T, we may interpret the elements in T as those invertible elements in $\mathbb{Z}/e'\mathbb{Z}$ that are of the form $1 + kd$ for some k because ...

227. (265) **Katz, N. M. (1977)**, ‘The Eisenstein measure and p-adic interpolation’. *American Journal of Mathematics*, volume **99**, pages 238–311.

[p. 273. Lines 4–2 from bottom.]

What about special values? By construction, we have, for $k \geq 1$,

$$\mathcal{L}_{(E,\lambda\omega)}\left(\chi_k\right) = 2J_k\left(E,\lambda\omega\right) = 2G_k\left(E,\lambda\omega\right) - 2p^{k-1}\left(\mathrm{Frob}G_k\right)\left(E,\lambda\omega\right),$$

a formula we will be able to unwind only in special cases.

228. (261) **Kaufman, R. (1974)**, 'Sets of multiplicity and differentiable functions. II'. *Transactions of the American Mathematical Society*, volume **200**, pages 427–435.

[p. 429. Lines 8–10.]

Because sets of small Lebesgue measure have small μ_k-measure, a proper choice of $Y = \eta_{k+1}$ enables us to obtain the necessary estimates.

229. (134, 189, 268) **Keinert, F. (1989)**, 'Inversion of k-plane transforms and applications in computer tomography'. *SIAM Review*, volume **31**, pages 273–298.

[p. 273. Lines 13–11 from bottom.]

Usually, a function f on $\mathbb{R}^2$ or $\mathbb{R}^3$ is determined from its integrals over lines or planes. Here f represents some internal property of an object which cannot be observed directly, while some integrals of f are observable.

230. (40, 56, 219) **Kellum, K. R. and H. Rosen (1992)**, 'Compositions of continuous functions and connected functions'. *Proceedings of the American Mathematical Society*, volume **115**, pages 145–149.

[p. 145. Lines 1–2.]

In calculus, we encounter the result that the composition $g \circ f$ of two continuous functions $f : I \to I$ and $g : I \to I$ is continuous, where $I = [0, 1]$.

231. (198) **Kendig, K. M. (1983)**, 'Algebra, geometry, and algebraic geometry: Some interconnections'. *American Mathematical Monthly*, volume **90**, pages 161–174.

[p. 166. Fourth paragraph.]

Parametrize $y = x^2$ by $x = t$, $y = t^2$, and plug into $Ax + By$.

232. (253) **Kierstead, H. A. (1981)**, 'An effective version of Dilworth's Theorem'. *Transactions of the American Mathematical Society*, volume **268**, pages 63–77.

[p. 63. Lines 11–9.]

Dilworth's Theorem [**D**] asserts than any partial ordering of finite width n can be covered with n chains. If the partial ordering is finite, then one can actually exhibit these chains (by trial and error, if by no other method).

233. (56, 127, 135) **Klambauer, G. (1978)**, 'Integration by parts and inverse functions'. *American Mathematical Monthly*, volume **85**, pages 668–669.

[p. 668. Line 1.]

Let f be a strictly increasing function with continuous derivative on a compact interval $[a, b]$.

234. (222) **Klebanoff, A. and J. Rickert (1998)**, 'Studying the cantor dust at the edge of feigenbaum diagrams'. *The College Mathematics Journal*, volume **29**, pages 189–198.

[p. 196. Second line below figure.]

The largest region leaves in one iteration and is bounded by the curves

$$x_{1_i} = \frac{1}{2} + (-1)^i \sqrt{\frac{1}{4} - \frac{1}{a}}, \qquad i = 1, 2$$

which satisfy $f_a(x) = 1$.

235. (269) **Kleiner, I. (1996)**, 'The genesis of the abstract ring concept'. *American Mathematical Monthly*, volume **103**, pages 417–424.

[p. 417. First two lines of paragraph (b).]

The polynomial rings $\mathbb{R}[x]$ and $\mathbb{R}[x, y]$ in one and two variables, respectively, share important properties but also differ in significant ways.

236. (93, 201) **Kleiner, I. (1999)**, 'Field theory: From equations to axiomatization'. *American Mathematical Monthly*, volume **106**, pages 677–684.

[p. 678. Lines 18–20.]

This says (in our terminology) that if E is the splitting field of a polynomial $f(x)$ over a field F, then $E = F(V)$ for some rational function V of the roots of $f(x)$.

237. (68, 238) **Klosinski, L. F., G. L. Alexanderson, and L. C. Larson (1993)**, ‘The fifty-third William Lowell Putnam Mathematical Competition’. *American Mathematical Monthly*, volume **100**, pages 755–767.

[p. 757. Problem A-2.]

Define $C(\alpha)$ to be the coefficient of x^{1992} in the power series expansion about $x = 0$ of $(1 + x)^{\alpha}$. Evaluate . . .

238. (76) **Klosinski, L. F., G. L. Alexanderson, and L. C. Larson (1993)**, 'The fifty-third William Lowell Putnam Mathematical Competition'. *American Mathematical Monthly*, volume **100**, pages 755–767.

[p. 758. Problem B-1.]

Let S be a set of n distinct real numbers.

239. (73, 101, 108, 179) **Knoebel, R. A. (1981)**, 'Exponentials reiterated'. *American Mathematical Monthly*, volume **88**, pages 235–252.

[p. 235. Lines 1–3.]

When is x^y less than y^x? For what kind of numbers does $x^y = y^x$? And is there a formula for y as a function of x?

240. (193) **Knuth, D. E. (1981)**, 'A permanent inequality'. *American Mathematical Monthly*, volume **88**, pages 731–740.

[p. 732. Lines 2–7.]

For example, the quadratic form $x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_2x_3$ can be specified either by the triangular matrix

$$\begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

or by the symmetric matrix

$$\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

241. Kolman, B., R. C. Busby, and S. Ross (1996), *Discrete Mathematical Structures, 3rd Edition.* Prentice-Hall.

[p. 111. Theorem 2.]

Let R and S be relations from A to B. If $R(a) = S(a)$ for all a in A, then $R = S$.

242. (68, 185) **Konvalina, J. (2000)**, 'A unified interpretation of the binomial coefficients, the Stirling numbers, and the Gaussian coefficients'. *American Mathematical Monthly*, volume **107**, pages 901–910.

[p. 902. Lines 2–4.]

Define *the binomial coefficient of the first kind* $\left(\begin{array}{c} n \\ k \end{array} \right)$ to be the number of k-element subsets of S; that is, the number of ways to choose k distinct objects from S with the order of selection not important.

243. (34, 126) **Kong, T. Y., R. Kopperman, and P. R. Meyer (1991)**, 'A topological approach to digital topology'. *American Mathematical Monthly*, volume **98**, pages 901–917.

[p. 904. Lines 1–3.]

For $x \in X$ let $N(x)$ denote the intersection of all open neighborhoods of x, which of course is not a neighborhood in general.

244. (59, 201) **Kopperman, R. (1988)**, 'All topologies come from generalized metrics'. *American Mathematical Monthly*, volume **95**, pages 89–97.

[p. 93. Lines 9–11.]

If for each $i \in I$, A_i is a value semigroup (together with $+_i$, 0_i, ∞_i), then so is their product (with $+$, 0 defined coordinatewise; 1/2 and inf are also taken coordinatewise.)

245. (29, 208) **Krantz, S. G. (1995)**, *The Elements of Advanced Mathematics.* CRC Press. [p. 40. Proposition 3.1.]

246. (219) **Krantz, S. G. (1995)**, *The Elements of Advanced Mathematics.* CRC Press. [p. 57. Definition 4.16.]

247. (93, 110, 160) **Kreyszig, E. (1994)**, 'On the calculus of variations and its major influences on the mathematics of the first half of our century. part II'. *American Mathematical Monthly*, volume **101**, pages 902–908.

[p. 903. Last five lines.]

Dirichlet's Principle. There exists a function u that minimizes the functional (the so-called *Dirichlet Integral)*

$$D[u] = \int_\Omega |\operatorname{grad} u|^2 \, dV, \qquad \Omega \subset \mathbb{R}^2 \text{ or } \mathbb{R}^3,$$

among all functions $u \in C^1(\Omega) \cap C^0\left(\bar{\Omega}\right)$ which take on given values f on the boundary $\partial\Omega$ of Ω, ...

248. (191, 258) **Kulisch, U. W. and W. L. Miranker (1986)**, 'The arithmetic of the digital computer: A new approach'. *SIAM Review*, volume **28**, pages 1–40.

[p. 36. Lines 8–9.]

We make use of a type concept and an operator concept as well as the overloading of (certain) function names.

249. (18, 117, 209) **Kupka, J. and K. Prikry (1984)**, 'The measurability of uncountable unions'. *American Mathematical Monthly*, volume **91**, pages 85–97.

[p. 86. Lines 5–6.]

Conversely, if an arbitrary function $f : X \to \mathbb{R}$ satisfies (1.6) and if the (finite Radon) measure μ is complete, then f is $\mathcal{A}$-measurable.

250. (185) **Lam, C. W. H. (1991)**, 'The search for a finite projective plane of order 10'. *American Mathematical Monthly*, volume **98**, pages 305–318.

[p. 305. Lines 11–12.]

A *finite projective plane of order* n, with $n > 0$, is a collection of $n^2 + n + 1$ lines and $n^2 + n + 1$ points such that ...

251. (29, 73, 122) **Lang, S. (1965)**, *Algebra.* Addison Wes-ley.

[p. 56. Lines 9–2 from bottom.]

Let A be a ring, and let U be the set of elements of A which have both a right and a left inverse. ... Therefore U satisfies all the axioms of a multiplicative group, and is called the group of *units* of A. It is sometimes denoted by A^*, and is also called the group of *invertible elements* of A.

252. (98, 108, 120, 135, 150, 192) **Lance, T. and E. Thomas (1991)**, 'Arcs with positive measure and a space-filling curve'. *American Mathematical Monthly*, volume **98**, pages 124–127.

[p. 126. Lines 10–4 from bottom.]

Fix some $s \in (0,1]$, and consider the Cantor set C_s obtained by removing an open interval of length $(1-s)/2$ from the middle of I, open intervals of length $(1-s)/8$ from the center of the two resulting open intervals, open intervals of length $(1-s)/32$ from the four resulting intervals, and so on. The Cantor set C_s has one dimensional Lebesgue measure s. There is a standard mapping of C_s onto I, the Cantor function $\Phi_s \ldots$ which can be embedded as the last stage of a one parameter family of maps $t \to \Phi_{s,t}$.

253. (124, 210, 212) **Lau, A. T.-M. (1081)**, 'The second conjugate algebra of the Fourier algebra of a locally compact group'. *Transactions of the American Mathematical Society*, volume **267**, pages 53–63.

[p. 55. Proposition 3.2.]

PROPOSITION 3.2. (a) $VN(G)^*$ has a right identity if and only if G is amenable. (b) $VN(G)^*$ has a left identity if and only if G is compact.

254. (148, 201, 276) **Lawlor, G. (1996)**, 'A new minimization proof for the brachistochrone'. *American Mathematical Monthly*, volume **103**, pages 242–249.

[p. 243. Proposition 1.2.]

The velocity of a marble rolling without friction down a ramp is proportional to $\sqrt{|y|}$*, if the marble starts at rest at a point where* $y = 0$*.*

255. (131, 240) **Leary, F. C. (1989)**, ‘Rings with invertible regular elements’. *American Mathematical Monthly*, volume **96**, pages 924–926.

[p. 924. Lines 6–5 from bottom.]

The key ingredient in the preceding discussion is the fact that for finite sets S, every injective $f : S \to S$ is necessarily surjective.

256. (209) **Leep, D. B. and G. Myerson (1999)**, 'Marriage, magic and solitaire'. *American Mathematical Monthly*, volume **106**, pages 419–429.

[p. 428. Corollary 13.]

Corollary 13. No vector space V over an infinite field F is a finite union of proper subspaces.

257. (118, 118, 143, 189) **Lefton, P. (1977)**, 'Galois resolvents of permutation groups'. *American Mathematical Monthly*, volume **84**, pages 642–644.

[p. 643. Lines 1–2.]

DEFINITION. Let $\Phi(z, y)$ be the minimal polynomial for $F(x)$ over $Q(y)$. We call $\Phi(z, y)$ the Galois resolvent of Π corresponding to $F(x)$.

258. (248) **Lehning, H. (1990)**, 'Computer-aided or analytic proof?'. *The College Mathematics Journal*, volume **21**, pages 228–239.

[p. 228. Lines 3–5.]

My answer was 23124110, and the story might have ended there; but though I do not know why, I continued: 1413223110. 1423224110, 2413323110, 1433223110, 1433223110, and then the sequence is constant from this term on.

259. (115, 255) **Lenstra, Jr., H. W. (1992)**, 'Algorithms in algebraic number theory'. *Bulletin of the American Mathematical Society (N.S.)*, volume **26**, pages 211–244.

[p. 216. Lines 21–23.]

Other algorithms may even give a nontrivial factor of p, . . .

260. (140) **Lewin, J. (1991)**, 'A simple proof of Zorn's Lemma'. *American Mathematical Monthly*, volume **09**, pages 353–354.

[p. 353. Title.]

A Simple Proof of Zorn's Lemma.

261. (25, 34, 69, 93, 123) **Lewis, H. R. and C. H. Papadimitriou (1998)**, *Elements of the Theory of Computation, 2nd Edition*. Prentice-Hall.

[p. 20. Lines 11–10 from the bottom.]

We call two sets A and B **equinumerous** if there is a bijection $f : A \to B$. [This statement also occurs on page 21 of the First Edition.]

262. (127) **Lew, J. S. and J. Donald A. Quarles (1989)**, 'Optimal inscribed polygons in convex curves'. *American Mathematical Monthly*, volume **96**, pages 886–902.

[p. 886. Lines 1–3.]

For a given positive integer n, and a regular C^2 plane convex curve Γ that includes no straight line segment, we wish to approximate Γ by in inscribed n-segment polygonal line that minimizes the area between curve and polygonal line.

263. (81) **Lidl, R. and G. Pilz (1984)**, *Applied Abstract Algebra.* Springer-Verlag. [p. xv. Line 5.]

264. (148) **Llewellyn, D. C., C. Tovey, and M. Trick (1988)**, 'Finding saddlepoints of two-person, zero sum games'. *American Mathematical Monthly*, volume **95**, pages 912–918.

[p. 913. Lines 19–18 from bottom.]

For ease of notation, we will denote $\log_2 x$ by $\lg x$.

265. (31, 125) **Loeb, P. A. (1991)**, 'A note on Dixon's proof of Cauchy's Integral Theorem'. *American Mathematical Monthly*, volume **98**, pages 242–244.

[p. 243. Lines 4–5.]

The trace of the curve γ in the complex plane is denoted by $\{\gamma\}$.

266. (34) **Lorch, E. R. (1971)**, 'Continuity and Baire functions'. *American Mathematical Monthly*, volume **78**, pages 748–762.

[p. 753. Item (2) under **III**.]

There are three cases:

(1) $\boldsymbol{E}$ has finite cardinality.

(2) $\boldsymbol{E}$ has denumerable cardinality.

(3) $\boldsymbol{E}$ has the cardinality c of the continuum.

267. (34) **St. Luke (1949)**, 'The Gospel according to St. Luke'. In *The Holy Bible, Authorized King James Version*, pages 55–89. Collins Clear-Type Press.

[p. 56. Chapter 1, verse 48.]

. . . from henceforth all generations shall call me blessed.

268. (91) **Lyzzaik, A. K. and K. Stephenson (1991)**, 'The structure of open continuous mappings having two valences'. *Transactions of the American Mathematical Society*, volume **327**, pages 525–566.

[p. 547. Lines 9–11.]

Our objective in the remainder of this section is to develop the $\mathcal{U}$-homotopy necessary to replace a general, perhaps pathological, function f by a more manageable and regular function h.

269. (250) **Mac Lane, S. (1971)**, *Categories for the Working Mathematician.* Springer-Verlag.

[p. 58. Line 15.]

Thus a universal element $\langle r, e\rangle$ for H is exactly a universal arrow from $*$ to H.

270. (7) **Mac Lane, S. (1981)**, 'Mathematical models: A sketch for the philosophy of mathematics'. *American Mathematical Monthly*, volume , pages 462–472.

[p. 471. Lines 5–6.]

Abstraction consists in formulating essential aspects of some subject matter in terms of suitable axioms.

271. (147, 172) **MacEachern, S. N. and L. M. Berliner (1993)**, 'Aperiodic chaotic orbit'. *American Mathematical Monthly*, volume **100**, pages 237–241.

[p. 237. Lines 12–16.]

For ease of exposition, the presentation here is specialized to a familiar example for $J = [0, 1]$, namely the tent map defined by the function

$$\begin{aligned} f(x) = 2x &\qquad 0 \leq x < .5 \\ 2 - 2x &\qquad .5 \leq x \leq 1 \end{aligned}$$

272. (172) **MacEachern, S. N. and L. M. Berliner (1993)**, 'Aperiodic chaotic orbit'. *American Mathematical Monthly*, volume **100**, pages 237–241.

[p. 241. Lines 13–11 from bottom.]

For any $y \in S_1$, witn $y \neq 1$, there are exactly two values of z (namely $z = y/2$ or $1 - y/2$) for which $\sigma(\mathbf{t}_z) = \mathbf{t}_y$.

273. (24, 126, 131, 183, 261) **Mac Lane, S. and G. Birkhoff (1993)**, *Algebra, third edition.* Chelsea.

[p. 43. Lines 5–12.]

A *group* is a set G together with a binary operation $G \times G \to G$, written $(a, b) \mapsto ab$, such that: ... In other words, a group is a monoid in which every element is invertible.

274. (265) **Mac Lane, S. and G. Birkhoff (1993)**, *Algebra, third edition.* Chelsea.

[p. 182. Lines 4–3 from the bottom.]

Therefore these identities characterize the biproduct $A_1 \oplus A_2$ up to isomorphism ...

275. (72, 168, 182, 201, 232, 276) **Macki, J. W., P. Nistri, and P. Zecca (1993)**, 'Mathematical models for hysteresis'. *SIAM Review*, volume **35**, pages 94–123.

[p. 98. Beginning of Section 2.3.]

The Preisach model of electromagnetic hysteresis dates from 1935 ... This model uses a superposition of especially simple independent relay hysteresis operators ... That is,

$$F\left[v\right]\left(t\right)=\int\int\mu\left(\alpha,\beta\right)\hat{F}_{\alpha,]beta}\left[v\right]\left(t\right)d\alpha d\beta$$

where $\mu(\alpha,\beta)\geq 0$ is a weight function, usually with support on a bounded set in the (α,β)-plane, $\hat{F}_{\alpha\beta}$ is a relay hysteresis operator with thresholds $\alpha<\beta$, and

$$\begin{aligned} h_U(v) &= +1 \quad \text{on } \left[\alpha,\infty\right); \\ h_L(v) &= -1 \quad \text{on } \left(-\infty,\beta\right]. \end{aligned}$$

276. (125, 269) **Manoharan, P. (1995)**, 'Generalized Swan's theorem and its application'. *Proceedings of the American Mathematical Society*, volume **123**, pages 3219–3223.

[p. 3219. Last four lines of abstract.]

A characterization for all the smooth maps between the spaces of vector bundles, whose kth derivatives are linear differential operators of degree r in each variable, is given in terms of $A^{(r)}$ maps.

277. (31, 32) **Marx, I. (1962)**, 'Transformation of series by a variant of Stirling's numbers'. *American Mathematical Monthly*, volume **69**, pages 530–532.

[p. 530. Lines 10–9.]

In terms of this function, "bracket symbols" $\left[{n \atop j} \right]$ and "brace symbols" $\left\{ {n \atop i} \right\}$ may be defined by the formulas ...

278. (203) **Richard F. Maruszewski, J. (1991)**, 'Programs for a logic course'. *The College Mathematics Journal*, volume **22**, pages 235–241.

[p. 236. Lines 4–8.]

... we use the weather outlook for the week as the basis for our world. We might write, for instance,

weather(sunday,fair).

Here we are representing the fact, "the weather on Sunday will be fair" by a predicate, weather, with two arguments.

279. (34, 76) **Marchisotto, E. A. (1992)**, 'Lines without order'. *American Mathematical Monthly*, volume **99**, pages 738–745.

[p. 739. Lines 23–25.]

Given two distinct points a and b, the class of all points p such that there exists a motion that leaves a fixed and transforms p into b is called a *sphere* of center a and passing through b.

280. (98) **Marchisotto, E. A. (1992)**, 'Lines without order'. *American Mathematical Monthly*, volume **99**, pages 738–745.

[p. 741. Lines 2–4.]

Notice that the rotation in Euclidean space described above not only fixes a and b, but also fixes all points collinear with a and b.

281. (35, 107) **Martelli, M., M. Deng, and T. Seph (1998)**, 'Defining chaos'. *Mathematics Magazine*, volume **72**, pages 112–122.

[p. 116. Example 3.1.]

Let $f : [0,1] \to [0,1]$ be defined by

$$f(x) = \begin{cases} x + .5 & 0 \leq x \leq .5 \\ 0 & .5 \leq x \leq 1. \end{cases}$$

282. (21, 177) **Matúš, F. (94)**, 'On nonnegativity of symmetric polynomials'. *American Mathematical Monthly*, volume **101**, pages 661–664.

[p. 661. Lines 8–4.]

Theorem. Let $x \in \mathbb{C}^N$ and let the values $p_{[\alpha]}(x)$ be nonnegative for all $\alpha \in \mathcal{A}$. Then necessarily all coordinates x_i, $i \in N$, are nonnegative.

283. (22, 126, 222) **Mauldon, J. G. (1978)**, ‘Num, a variant of Nim with no first-player win’. *American Mathematical Monthly*, volume **85**, pages 575–578.

[p. 575. Lines 3–5.]

These constraints are such that, if one player could in his turn leave (say) n matchsticks in a particular heap, then the other player could not. In particular, at most one of the players is entitled to clear any particular heap.

284. (258) **Maxson, C. J. (1003)**, 'Near-rings of invariants. II'. *Proceedings of the American Mathematical Society*, volume **117**, pages 27–35.

[p. 27. Line 4.]

Under function addition, $(M, +)$ is a group.

285. (122, 147, 178) **Mazur, B. (1993)**, 'On the passage from local to global in number theory'. *Bulletin of the American Mathematical Society (N.S.)*, volume **29**, pages 14–50.

[p. 29. Lines 8–6 from the bottom.]

From now on we assume n to be good, and we will freely identify elements of $H^1(G_K, E[n])$ with the Δ_n-equivariant homomorphisms $G_M \to E[n]$ to which they give rise.

286. (236, 242, 268) **Hill, R. J. and M. S. Mazur (1995)**, 'The matrix exponential function and systems of differential equations using Derive'. *The College Mathematics Journal*, volume **26**, pages 146–151.

[p. 147. Lines 7–11.]

Perhaps the greatest advantage to the use of symbolic computation is that a student can work with relatively complicated examples without losing sight of the process for finding the solution. Such a method takes full advantage of the capacity of the software to substitute matrices into scalar polynomials, to perform integration, and to simplify matrix expressions.

287. (202) **McAllister, B. L. (1966)**, 'Embedding an integral domain using cyclic elements'. *American Mathematical Monthly*, volume **73**, pages 994–995.

[p. 994. Lines 9–11.]

Let P denote the set of *all* ordered pairs of elements of D, and define the relation R over P by

$$(a, b)R(c, d) \Leftrightarrow ad = bc$$

288. (89) **McCarthy, P. J. (1960)**, 'Some remarks on arithmetical identities'. *American Mathematical Monthly*, volume **67**, pages 539–548.

[p. 547. Lines 17–18.]

Consequently, in order to evaluate $f(M,N)$ we need only evaluate $f\left(p^a,p^b\right)$ where p is a prime.

289. (118, 198, 219, 248, 266) **McColm, G. L. (1989)**, 'Some restrictions on simple fixed points of the integers'. *Journal of Symbolic Logic*, volume **54**, pages 1324–1345.

[p. 1328. Lines 10–13.]

Simultaneously, we define the *evaluation function*

$$\mathrm{ev} : \mathrm{terms} \times \mathrm{assignments} \to \mathrm{values}$$

so that if θ is a term and $\mathbf{x}$ is an assignment of elements from ω and perhaps partial functions on ω, then $\mathrm{ev}(\theta, \mathbf{x})$ is the result of plugging $\mathbf{x}$ into θ.

290. (166, 200) **Mckenzie, R. (1975)**, 'On spectra, and the negative solution of the decision problem for identities having a finite nontrivial model'. *Journal of Symbolic Logic*, volume **40**, pages 186–196.

[p. 187. First sentence of fifth paragraph.]

We use Polish notation (sometimes with added parentheses for clarity) to denote the *terms* build up from function symbols and variables of a first order language.

291. (272) **McKenzie, R. (1971)**, 'Negative solution of the decision problem for sentences true in every subalgebra of $\langle N, +\rangle$'. *Journal of Symbolic Logic*, volume **36**, pages 6007–609.

[p. 608. Lines 19–21.]

In order to ensure that no undesirable clash of variables occurs in what follows, we assume that the variables of the language of K are listed without repetition as $x, y, u_0, u_1, u_2, z_0, z_2, z_2, x_0, x_1, x_2, \ldots$.

292. (93, 134) **Mead, D. G. (1961)**, 'Integration'. *Mathematical Association of America*, volume **68**, pages 152–156.

[p. 152. Lines 9–12.]

With respect to integration, however, he is well aware of his rather unhappy position, that of knowing that some functions cannot be integrated (in finite terms) but of possessing no way of determining whether a particular function is in this class or not.

293. (17, 50, 93, 118, 126, 239) **Mead, D. G. (1993)**, 'Generators for the algebra of symmetric polynomials'. *American Mathematical Monthly*, volume **100**, pages 386–388.

[p. 387. Lines 12–16.]

Consider the monomial symmetric function $\langle a_1, a_2, \ldots, a_k \rangle$ *in* $Q[x_1, x_2, \ldots, x_n]$ *and let* $t = \Sigma_{i=1}^{k} a_i$. *Then there is a positive rational number* c *and an element* B *in* $Q[p_1, p_2, \ldots, p_{i-1}]$ *such that*

$$\langle a_1, a_2, \ldots, a_k \rangle = (-1)^{k_1} c p_i + B.$$

294. (170) **Melissen, H. J. B. M. (1993)**, 'Densest packings of congruent circles in an equilateral triangle'. *American Mathematical Monthly*, volume **100**, pages 916–925.

[p. 917. Lines 1–4 of Section 2.1.]

Two of the four points must lie in the same subregion from the partition shown in Figure 3a, so $d_4 \leq \frac{1}{\sqrt{3}}$. If the upper bound is attained, then one point must lie at the center and the other onr is a vertex of the triangle.

295. (170) **Melissen, H. (1997)**, 'Loosest circle coverings of an equilateral triangle'. *Mathematics Magazine*, volume **70**, pages 118–124.

[p. 119. Lines 8–7 from bottom.]

If the vertices of the triangle are covered by two discs, one of the discs must cover two vertices, ...

296. (82) **Mendelson, E. (1987)**, *Introduction to Mathematical Logic*. Wadsworth and Brooks/Cole. [p. 3. Line 13.]

297. (21, 172, 176) **Meyers, L. F. (1959)**, 'An integer construction problem'. *American Mathematical Monthly*, volume **66**, pages 556–561.

[p. 560. Beginning of section 5.]

In this section two additional operations, namely subtraction and negation, will be allowed. For example the combination (-3) involves negation but not subtraction.

298. (131, 143, 150) **Meyer, M. J. (1992)**, 'Continuous dense embeddings of strong Moore algebras'. *Proceedings of the American Mathematical Society*, volume **116**, pages 727–735.

[p. 732. Lines 5–8.]

Since the inverse images of hulls are hulls, the map $Q \in \mathrm{Prim}\,(\mathcal{B}) \to \phi^{-1}(Q) \in \mathrm{Prim}\,(\mathcal{A})$ is continuous. It remains to be shown that it is one to one. Suppose that N, $Q \in \mathrm{Prim}\,(\mathcal{B})$ are such that $\phi^{-1}(N) = \phi^{-1}(Q)$. Choose continuous irreducible representations π, σ of $\mathcal{B}$ on Banach spaces X, Y such that $N = \ker(\pi)$ and $Q = \ker(\sigma)$.

299. (110, 276) **Michal, A. D. (1950)**, 'Integral equations and functionals'. *Mathematics Magazine*, volume **24**, pages 83–95.

[p. 86–87. Lines 3–1 from bottom.]

Definition of a functional. *A functional is a function on* C_1 *to* $C - 2$ *whenever* C_1 *is itself a class of functions of a finite number of numerical variables whicle* C_2 *is another class of functions of a finite number of numerical variables.*

300. (139) **Milton, E. O. (1972)**, 'Asymptotic behavior of transforms of distributions'. *Transactions of the American Mathematical Society*, volume **172**, pages 161–176.

[p. 170. Lines 6–4 above Theorem 4.3.]

In the preceding initial value type theorem the distributions and measures were restricted to have support in $[0, \infty)$ by the growth of $e^{-\sigma x}$ for negative x and large positive σ.

301. (117, 131) **Millar, T. (1989)**, 'Tame theories with hyperarithmetic homogeneous models'. *Proceedings of the American Mathematical Society*, volume **105**, pages 712–746.

[p. 714. Line 4 from bottom.]

The c_ξ's inhabit distinct equivalence classes:

302. (50, 107, 184, 209, 222, 266, 268) **Mollin, R. A. (1997)**, 'Prime-producing quadratics'. *American Mathematical Monthly*, volume **104**, pages 529–544.

[p. 531. Definition 2.1.]

Consider $F(x) = ax^2 + bx + c$ *(*$a, b, c \in \mathbb{Z}$*),* $a \neq 0$*, and suppose* $|F(x)|$ *is prime for all integers* $= 0, 1, \ldots, l-1$*. If* $l \in \mathbb{N}$ *is the* smallest *value such that* $|F(l)|$ *is composite,* $|F(l)| = 1$*, or* $|F(l)| = |F(x)|$ *for some* $x = 0, 1, \ldots, l-1$*, then* $F(x)$ *is said to have* prime-production length l.

303. (136, 138, 198, 211, 240) **Morgan, F. (1988)**, 'Area-minimizing surfaces, faces of Grassmannians, and calibrations'. *American Mathematical Monthly*, volume **95**, pages 813–822.

[p. 814. Lines 2–5 below first figure.]

The coordinates of the line at angle θ to the x-axis are the oriented projections of a unit length from that line onto the axes, namely $(\cos\theta, \sin\theta)$. Hence $G(1, \mathbb{R}^2)$ is just the unit circle in $\mathbb{R}^2$.

304. (105) **Morrison, K. E. (1995)**, 'Cosine products, Fourier transforms, and random sums'. *American Mathematical Monthly*, volume **102**, pages 716–724.

[p. 716. Line 1.]

The function $\sin x/x$ is endlessly fascinating.

305. (166) **Morrison, K. E. (1995)**, 'Cosine products, Fourier transforms, and random sums'. *American Mathematical Monthly*, volume **102**, pages 716–724.

[p. 720. Lines 12–15.]

Flip a fair coin repeatedly. Beginning with 0, add 1/2 if the result is heads and subtract 1/2 if the result is tails. On the next toss add or subtract 1/4; on the next add or subtract 1/8, and so on.

306. (18) **Mornhinweg, D., D. B. Shapiro, and K. G. Valente (1993)**, 'The principal axis theorem over arbitrary fields'. *American Mathematical Monthly*, volume **100**, pages 749–754.

[p. 749. Title of Article.]

The Principal Axis Theorem Over Arbitrary Fields

307. (99, 261) **Mornhinweg, D., D. B. Shapiro, and K. G. Valente (1993)**, 'The principal axis theorem over arbitrary fields'. *American Mathematical Monthly*, volume **100**, pages 749–754.

[p. 751. Theorem 2.]

Let F be a formally real pythagorean field. The following are equivalent:
(i) F has the Principal Axis Property,
(ii) Every symmetric matrix over F is diagonalizable over F, and
(iii) Every symmetric matrix over F has an eigenvalue in F.

308. (86, 192) **Neidinger, R. D. and R. J. Annen III (1996)**, 'The road to chaos is filled with polynomial curves'. *American Mathematical Monthly*, volume **103**, pages 640–653.

[p. 642. Lines 15–23.]

Superattracting Root Theorem. *Let* $n \in \mathbf{N}$*. The parameter* r *satisfies* $Q_n(r) = 0$ *and* $Q_j(r) \neq 0$ *for* $0 < j < n$ *if and only if iteration of* $f_r(x)$ *has a superattracting periodic point of period* n*.*

309. (172) **Nevo, A. (1994)**, 'Harmonic analysis and pointwise ergodic theorems for noncommuting transformations'. *Journal of the American Mathematical Society*, volume **7**, pages 875–902.

[p. 875. Lines 10–11.]

Let $(X, \mathcal{B}, m)$ be a standard Lebesgue measure space, namely a measure space whose σ-algebra is countably generated and countably separate.

310. (97, 117, 120) **Newns, W. F. (1967)**, 'Functional dependence'. *American Mathematical Monthly*, volume **74**, pages 911–920.

[p. 911. Lines 13–12 from bottom.]

Let X, $\mathbf{I}$ be sets, $(Y_\iota)_{\iota\in\mathbf{I}}$ a family of sets, and for each $\iota \in \mathbf{I}$ let $f_\iota : X \to Y_\iota$.

311. (175, 204) **Newcomb, S. (1881)**, 'Note on the frequency of use of the different digits in natural numbers'. *American Journal of Mathematics*, volume **4**, pages 39–40.

[p. 39. Lines 8–10, lines 2–1 from the bottom, and lines 1–2 on page 40.]

The question we have to consider is, what is the probability that if a natural number be taken at random its first significant digit will be n, its second n', etc. . . .

Our problem is thus reduced to the following:

We have a series of numbers between 1 and i, . . .

312. (148, 268) **Newns, W. F. (1967)**, 'Functional dependence'. *American Mathematical Monthly*, volume **74**, pages 911–920.

[p. 912. Lines 3–4.]

The support of F is the smallest closed set outside which F vanishes identically.

313. (39, 61, 219, 222, 267) **Niven, I. (1956)**, *Irrational Numbers.* Mathematical Association of America.

[p. 41. Corollary 3.12.]

If θ *is rational in degrees, say* $\theta = 2\pi r$ *for some rational number* r*, then the only rational values of the trigonometric functions of* θ *are as follows:* $\sin\theta, \cos\theta = 0, \pm\frac{1}{2}, \pm 1$; $\sec\theta, \csc\theta = \pm 1, \pm 2$; $\tan\theta, \cot\theta = 0, \pm 1$.

314. (37, 189, 261) **Niven, I. (1956)**, *Irrational Numbers.* Mathematical Association of America.

[p. 83. Lemma 7.1.]

Any $r + 1$ linear forms in r indeterminates with rational coefficients are linearly dependent over the rationals.

315. (9, 211) **Novak, J. (1950)**, 'On a problem of E. Cech'. *Proceedings of the American Mathematical Society*, volume **1**, pages 211–214.

[p. 212. Lines 15–12 from bottom.]

Such cardinals do exist; in particular, taking R to be the set of rational numbers, well-ordering the irrationals, and defintion P_λ to be a sequence of rationals converging to the λth irrational, shows that $\aleph_0$ is such a cardinal.

316. (47, 160, 242) **Ogilvy, C. S. (1960)**, 'Exceptional extremum problems'. *American Mathematical Monthly*, volume **67**, pages 270–275.

[p. 70. Lines 9–6 from bottom.]

If we let x be the length of the new fence which is to be aligned with the original 100 feet and proceed in the usual fashion to maximize the resulting expression for area, x comes out to be -25, which is not permitted by the conditions of the problem.

317. (34, 39, 118, 267) **Osofsky, B. L. (1994)**, 'Noether Lasker primary decomposition revisited'. *American Mathematical Monthly*, volume **101**, pages 759–768.

[p. 760. Lines 14–16.]

With this convention on sides, the defining property of a module homomorphism $\phi : M \to N$ is that $(r \cdot x + x \cdot y)\phi = r \cdot (x)\phi_s \cdot (y)\phi$ for all $x, y \in M$ and $r, s \in R$.

318. (124) **Osofsky, B. L. (1994)**, 'Noether Lasker primary decomposition revisited'. *American Mathematical Monthly*, volume **101**, pages 759–768.

[p. 766. Lines 14–17.]

Definition. If R is a commutative Noetherian ring and P is a prime ideal of R, then P is *associated with* the module M provided $P = (0 : x)$ for some $x \in M$. We will call M P-*primary* if the prime ideal P is associated with M and no other prime is.

319. (19) **Osserman, R. (1979)**, 'Bonnesen-style isoperimetric inequalities'. *American Mathematical Monthly*, volume **86**, pages 1–29.

[p. 5. Line 10.]

Before proceeding further with the argument, let us prove (27).

320. (118) **Osserman, R. (1979)**, 'Bonnesen-style isoperimetric inequalities'. *American Mathematical Monthly*, volume **86**, pages 1–29.

[p. 18. Lines 9–7 from bottom.]

The isoperimetric inequality (36) gives

$$L^2 \geq 4\pi A + \alpha^2 A^2 > \alpha^2 A^2$$

for simply connected domains with $K \leq -\alpha^2$, so that (77) holds in that case.

321. (192, 274) **Osserman, R. (1990)**, 'Curvature in the eighties'. *American Mathematical Monthly*, volume **97**, pages 731–756.

[p. 732. Lines 13–8 from bottom.]

Again using the parameter s of arc length along the curve, and denoting by x the position vector on the curve, we have the unit tangent vector

$$T = \frac{dx}{ds}$$

and the curvature k defined by

$$k = \left|\frac{dT}{ds}\right| = \left|\frac{d^2x}{ds^2}\right|.$$

322. (118, 118, 125, 261) **Ostrowski, A. M. (1971)**, 'Some properties of reduced polynomial equations'. *SIAM Journal on Numerical Analysis*, volume **8**, pages 623–638.

[p. 624. Lines 14–16.]

Taking $x = 2$ in (6), we obtain $\psi_n(2) = 1 + 2^{n-1}$, so that

$$(7) \qquad \rho_n < 2 \quad (n \geq 2)$$

and we see that all roots of a reduced equation lie in the disk $|z| < 2$.

323. (75, 97, 266) **Oxtoby, J. C. (1977)**, 'Diameters of arcs and the gerrymandering problem'. *American Mathematical Monthly*, volume **84**, pages 155–162.

[p. 155. Lines 24–25.]

For what values of c (if any) is it true that for every finite family of disjoint finite sets F_j with diam $F_j > 0$ there exist disjoint polygonal arcs A_i such that $F_j \subseteq A_i$ and diam $A_i \leq c$ diam F_j for all j?

324. (188) **Paakki, J., A. Karhinen, and T. Silander (1990)**, 'Orthogonal type extensions and reductions'. *ACM SIGPLAN Notices*, volume **25**, pages 28–38.

[p. Abstract. First two sentences.]

In this paper we present a generalization of Oberon's record type extensions. Our extension mechanism is orthogonally applicable to all the conventional data types found in Pascal-like languages.

325. (142, 204) **Pamfilos, P. and A. Thoma (1999)**, 'Appolonian cubics: An application of group theory to a problem in Euclidean geometry'. *Mathematics Magazine*, volume **72**, pages 356–366.

[p. 362. Lines 22–25.]

THEOREM 4 (VAN REES). *Let* $c = AP[AD, BC]$ *be an irreducible Appolonian cubic, let* A' *and* B' *be two arbitrary regular points on the cubic, and let* $C' = B' + O_1$, $D' = A' + O_1$. *Then the two Appolonian cubics* $AP[AD, BC]$ *and* $AP[A'D', B'C']$ *are identical.*

326. (268) **Dave Pandres, J. (1957)**, 'On higher ordered differentiation'. *American Mathematical Monthly*, volume **64**, pages 566–572.

[p. 566. Lines 1–4.]

Courant in [1] gives Lebniz' rule for finding the Nth derivative of the product of two functions, and then remarks that no such easily remembered law has been found for the repeated differentiation of the compound function $Y = F[U(X)]$.

327. (40, 268) **Parusinski, A. and P. Pragacz (1995)**, 'Chern-Schwartz-Macpherson classes and the Euler characteristic of degeneracy loci and special divisors'. *Journal of the American Mathematical Society*, volume **8**, pages 793–817.

[p. 803. Lines 3–5.]

We put $F_{v,w} = \mathcal{S}$, $E_{v,w} = \mathcal{R}_{X_{v,w}}$, and define $\phi_{v,w}$ as the composite:

$$F_{v,w} = \mathcal{S} \hookrightarrow (\mathcal{Q} \oplus \mathcal{R})_{X_{v,w}} \stackrel{\mathrm{pr}_2}{\to} E_{v,w} = \mathcal{R}_{X_{v,w}}.$$

328. (178, 221) **Pincus, J. D. (1964)**, 'On the spectral theory of singular integral operators'. *Transactions of the American Mathematical Society*, volume **113**, pages 101–128.

[p. 108. Lines 1–4.]

Therefore, we may deduce that

$$F(\xi, z) = \frac{T(\xi, z) - S(\xi, z)}{\sqrt{(S(\xi, z)T(\xi, z))}} \cdot \exp\left\{\frac{1}{2\pi i}\int_a^b \log\left|\frac{A(\mu) - \xi - \epsilon k(\mu)}{A(\mu) - \xi + \epsilon k(\mu)}\right| \frac{d\mu}{\mu - z}\right\}$$

and it is now clear that the roots of $F(\xi, z)$ are the roots of the function

$$W(\xi, z) = \frac{T(\xi, z) - S(\xi, z)}{\sqrt{(S(\xi, z)T(\xi, z))}}.$$

329. (209, 236) **Pólya, G. (1965)**, *Mathematical Discovery, Volume II.* John Wiley and Sons, Inc.

[p. 7. Lines 19–23.]

...from the last equation of sect. 7.4 we obtain

$$x = \frac{ah}{b - a}$$

Then we substitute this value for x in the two foregoing equations of sect 7.4, obtaining...

330. Pomerance, C. (1996), 'A tale of two sieves'. *Notices of the American Mathematical Society*, volume **43**, pages 1473–1485.

[p. 1482. Last sentence of second column.]

This discrepancy was due to fewer computers being used on the project and some "down time" while code for the final stages of the algorithm was being written.

331. (41, 177, 251) **Pomerance, C. (1996)**, 'A tale of two sieves'. *Notices of the American Mathematical Society*, volume **43**, pages 1473–1485.

[p. 1478. Lines 17–15 from the bottom of first column.]

If n is not a square modulo p, then $Q(x)$ is never divisible by p and no further computations with p need be done.

332. (58, 121) **Poor, H. V. (2000)**, 'Modulation and detection'. In *The Engineering Handbook*, Dorf, R. C., editor, page 4 of Chapter 126. CRC Press LLC.

[p. 4. Line 5.]

... where j denotes the imaginary unit.

333. (13, 73, 252) **Powers, R. T. (1974)**, 'Selfadjoint algebras of unbounded operator. II'. *Transactions of the American Mathematical Society*, volume **187**, pages 261–293.

[p. 264. Line 5.]

All algebras in this section will have a unit denoted by 1.

334. (51) **Powell, M. J. D. (1986)**, 'Convergence properties of algorithms for nonlinear optimization'. *SIAM Review*, volume **28**, pages 487–500.

[p. 490. Lines 4–8.]

3. Constrained optimization and the Maratos effect. In this section we consider the problem of calculating the least value of $\{\mathbf{F}(\mathbf{x})\,;\mathbf{x} \in \mathbb{R}^n\}$ subject to the constraints

$$(3.1) \qquad c_j(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, m,$$

on the variables. Usually there are some inequality constraints too, but the purpose of this section is achieved without this extra complication.

335. (38, 106, 194, 202, 238, 261, 267) **Powers, V. (1996)**, 'Hilbert's 17th problem and the champagne problem'. *American Mathematical Monthly*, volume **103**, pages 879–887.

[p. 879. Lines 1–4 and 20–21.]

About 15 years ago, E. Becker gave a talk in which he proved that

$$B(t) := \frac{1+t^2}{2+t^2} \in \mathbf{Q}(t))$$

is a sum of $2n$-th powers of elements in $\mathbf{Q}(t)$ for all n.

... A rational function $f \in \mathbf{R}(X) := \mathbf{R}(x_1, \ldots, x_k)$ is *positive semi-definite* (psd) if $f \geq 0$ at every point in $\mathbf{R}^k$ for which it is defined.

336. (226) **Prather, R. E. (1997)**, 'Regular expressions for program computations'. *American Mathematical Monthly*, volume **104**, pages 120–130.

[p. 123. Lines 22–18 from bottom.]

In effect, we thereby provide an elementary *operational semantics* for our program flowgraphs, saying that $L(F)$ is the "meaning" of F, describing as it does all possible sequences of elementary processes that could result. We note, however, that this is truly an "elementary" semantics, far less detailed than the *denotational semantics* that one ordinarily introduces in a programming language context.

337. (89, 268) **Prestrud, M. B. (1963)**, 'Hierarchic algebra'. *Mathematics Magazine*, volume **36**, pages 43–53.

[p. 43. Lines 1–3.]

Addition, subtraction, multiplication, division, exponentiation, or taking of roots, form an apparently tidy set of operations, but it is a set with some enticing anomalies.

338. (100, 127, 276) **Price, J. F. (1970)**, ‘Some strict inclusions between spaces of L^p-multipliers’. *Transactions of the American Mathematical Society*, volume **152**, pages 321–330.

[p. 321. Lines 6–8 of abstract.]

When $1 \leq p < q \leq 2$ or $2 \leq q < p \leq \infty$ it is known that

$$(1) \qquad L_p^p \subset L_q^q.$$

The main result of this paper is that the inclusion in (1) is strict unless G is finite.

339. (220, 251) **Price, J. J. (1989)**, 'Learning mathematics through writing: Some guidelines'. *The College Mathematics Journal*, volume **20**, pages 393–401.

[p. 394. Lines 10–12.]

The first time I gave the course, I asked the students not to submit their first drafts. But their attitude towards written homework was so ingrained that they would not even read their work, let alone revise it.

340. (29, 138) **Putnam, H. (1973)**, 'Recursive functions and hierarchies'. *American Mathematical Monthly*, volume **80**, pages 68–86.

[p. 82. Lines 1–6.]

If we adjoin to the above condition the further clause:

$$(Ex)(y)(z)\,(J(y,z) \in W_x \equiv J(y,z) \notin W_i)$$

then the definition becomes a definition of the class of recursive well-orderings (or, rather, of the corresponding set of indices), for this clause just says that the predicate W_i^2 has an r.e. complement $\overline{W}_x^2$, and a predicate is recursive just in case it and its complement are both r.e.

341. (20, 47, 87) **Quine, W. V. (1952)**, 'The problem of simplifying truth functions'. *American Mathematical Monthly*, volume **59**, pages 521–531.

[p. 521. Lines 6–10.]

One formula *implies* another if there is no assignment of truth values which makes the first formula true and the second false. Two formulas are *equivalent* if they imply each other. Implication and equivalence, so defined, are relations of formulas; they are not to be confused with the conditional and biconditional, commonly expressed by $\supset$ and $\Leftrightarrow$.

342. (31, 73, 214, 249, 262) **Rabinowitz, S. and P. Gilbert (1993)**, 'A nonlinear recurrence yielding binary digits'. *Mathematics Magazine*, volume **64**, pages 168–171.

[p. 168. Lines 10 and 3–1 from bottom.]

Let $\{x\}$ denote the fractional part of x, that is, $\{x\} = x - \lfloor x \rfloor$.

... 2. If k is an integer, a is a real number in the range $1 < a < 2$, and $x = k/(a-1)$, then

$$\left\lfloor a\lfloor x \rfloor + \frac{a}{2} \right\rfloor = \lfloor ax \rfloor$$

343. (71, 201, 261) **Ranum, D. L. (1995)**, 'On some applications of Fibonacci numbers'. *American Mathematical Monthly*, volume **102**, pages 640–645.

[p. 641. Lines 6–7 under Figure 2.]

At the other extreme, Figure 3 shows a worst case **degenerate** tree where each node has only 1 child except for the single leaf. [The trees here are binary trees.]

344. (118) **Reed, G. M. (1986)**, 'The intersection topology w.r.t. the real line and the countable ordinals'. *ransactions of the American Mathematical Society*, volume **297**, pages 509–520.

[p. 509. Lines 1–2.]

If Υ_1 and Υ_2 are topologies defined on the set X, then Υ is the *intersection topology* w.r.t. Υ_1 and Υ_2 defined on X, where $\{U_1 \cap U_2 \mid U_1 \in \Upsilon_1 \text{ and } U_2 \in \Upsilon_2\}$ is a basis for Υ.

345. (140, 239) **Reine, I. (1966)**, 'Nilpotent elements in rings of integral representations'. *Proceedings of the American Mathematical Society*, volume **17**, pages 270–274.

[p. 272. Lines 7–4.]

For any RG-module M of R-rank m, there is an exact sequence

$$0 \to U \otimes_R M \to RG \oplus \cdots \oplus RG \to \bar{R}^G \otimes_{\bar{R}} \bar{M} \to 0,$$

where m summands occur in the center module. This implies by Schanuel's Lemma that the module $U \otimes_R M$ depends only upon $\bar{M}$.

346. (25, 84, 131, 231, 255) **Ribet, K. A. (1995)**, 'Galois representations and modular forms'. *Bulletin of the American Mathematical Society (N.S.)*, volume **32**, pages 375–402.

[p. 391. Lines 12–13.]

Suppose that there is a non-trivial solution to Fermat's equation $X^\ell + Y^\ell = Z^\ell$.

347. (202, 209) **Ribenboim, P. (1996)**, 'Catalan's Conjecture'. *American Mathematical Monthly*, volume **103**, pages 529–538.

[p. 529. Lines 8–9.]

I may also consider the sequence of *all* proper powers, which includes the 5th powers, 7th powers, 11th powers, etc . . .

348. (227, 253) **Richert, N. (1992)**, 'Strang's strange figures'. *American Mathematical Monthly*, volume **99**, pages 101–107.

[p. 104. Lines 6–9.]

For x irrational, this defines a sequence $p_0/q_0, p_1/q_1, p_2/q_2 \dots$ of best approximations to x, with $q_0 < q_1 < q_1 < \dots$ In fact, an initial segment of the sequence can be calculated by trial and error from the definition simply by considering increasing q.

349. (188, 251) **Richmond, B. and T. Richmond (1993)**, 'The equal area zones property'. *American Mathematical Monthly*, volume **100**, pages 475–477.

[p. 475. Lines 7–8 below the figure.]

To state the problem precisely, suppose that $y = g(x)$ is a piecewise smooth nonnegative curve defined over $[a, b]$, and is revolved around the x-axis.

350. (62, 268) **Ringenberg, L. A. (1958)**, ‘Mathematics magazine’. *Mathematics Magazine*, volume **31**, pages 265–276.

[p. 265. Lines 15–12.]

The system of counting numbers consists of the elements 1, 2, 3, $\cdots$, and the fundamental operations (addition, subtraction, multiplication and division) for combining them.

351. (242, 269) **Hartley Rogers, J. (1963)**, ‘An example in mathematical logic’. *American Mathematical Monthly*, volume **70**, pages 929–945.

[p. 930. Lines 3–7.]

If we take variables and put them on either side of “=” or “<”, we obtain what we shall call *atomic formulas.* E.g.,

$$x = w, \ y < y, \ x_4 < z_2$$

are atomic formulas. These are the basic formulas (i.e., expressions) from which we shall build the larger formulas of our language.

352. Rosenthal, P. (1987), 'The remarkable theorem of Levy and Steinitz'. *American Mathematical Monthly*, volume **94**, pages 342–351.

[p. 342. Lines 9–11.]

The theorem is the following: *the set of all sums of rearrangements of a given series of complex numbers is the empty set, a single point, a line in the complex plane, or the whole complex plane.*

353. (48, 86, 128) **Rosen, K. (1993)**, *Elementary Number Theory and its Applications.* Addison-Wes-ley. [p. 208. Theorems 6.2 and 6.3.]

354. (86, 128, 276) **Rosen, K. (1993)**, *Elementary Number Theory and its Applications.* Addison-Wes-ley. [p. 223. Theorem 6.10..]

355. (29, 93, 219, 252) **Rosen, K. (1993)**, *Elementary Number Theory and its Applications.* Addison-Wes-ley. [p. 293. Lines 6–8.]

356. (37, 185) **Rosen, M. (1995)**, 'Niels Hendrik Abel and equations of the fifth degree'. *American Mathematical Monthly*, volume **102**, pages 495–505. [p. 504. Proposition 3.]

357. (108, 150, 222) **Ross, K. A. and C. R. B. Wright (1992)**, *Discrete Mathematics, 3rd Edition.* Prentice-Hall.

[p. 19. Lines 3–2 from bottom.]

We sometimes refer to a function as a **map** or **mapping** and say that f **maps** S into T.

358. (97) **Rota, G.-C. (1997)**, 'The many lives of lattice theory'. *Notices of the American Mathematical Society*, volume **44**, pages 1440–1445.

[p. 1440. .]

The family of all partitions of a set (also called equivalence relations) is a lattice when partitions are ordered by refinement.

359. (71, 125) **Roth, B. (1981)**, 'Rigid and flexible frameworks'. *American Mathematical Monthly*, volume **88**, pages 6–21.

[p. 12. First two lines of Example 4.2.]

Consider the degenerate triangle $G(p)$ in $\mathbb{R}^2$ shown in Fig. 4 with collinear vertices ...

360. (45, 178) **Rubel, L. A. (1989)**, 'The Editor's corner: Summability theory: A neglected tool of analysis'. *American Mathematical Monthly*, volume **96**, pages 421–423.

[p. 421. Lines 9–8 from bottom.]

We are now in a position to give a conceptual proof of Pringsheim's theorem ...

361. (67) **Saaty, T. L. (1972)**, 'Thirteen colorful variations on Guthrie's four-color conjecture'. *American Mathematical Monthly*, volume **79**, pages 2–43.

[p. 6. Lines 15–13 from bottom.]

1.2 DEFINITION: A k-**coloring** (or **proper** k-**coloring**) of a graph is an assignment of k colors to the vertices of the graph in such a way that no two adjacent vertices receive the same color.

362. (28, 106, 204) **Saari, D. G. (1990)**, 'A visit to the Newtonian n-body problem via elementary complex variables'. *American Mathematical Monthly*, volume **97**, pages 105–119.

[p. 111. Lines 10–5 from bottom.]

The equations of motion for w, when accompanied with the change of independent variables $ds = dt/r(t)$ introduced by Sundman, not only are well defined at $w = 0$, a collision, but they assume the particularly simple form

$$\mathbf{u}'' + a\mathbf{u} = 0$$

where a is some positive constant and $\mathbf{u} = (u_1, u_2)$ is the vector representation of w.

363. (51, 120) **Saari, D. G. and J. B. Urenko (1984)**, 'Newton's method, circle maps, and chaotic motion'. *American Mathematical Monthly*, volume **91**, pages 3–17.

[p. 5. Lines 20–19 from bottom.]

(2) the set $Z(f')$ is contained within the interval defined by the extreme points of $Z(f)$. A similar constraint holds for $Z(f'')$ with respect to the zeros of f'.

364. (120, 160) **Saerens, R. and W. R. Zame (1987)**, 'The isometry groups of manifolds and the automorphism groups of domains'. *Transactions of the American Mathematical Society*, volume **301**, pages 413–429.

[p. 413. Lines 1–3.]

Given an instance of a mathematical structure, we are led to ask for its group of symmetries; i.e., the group of structure-preserving self-transformations.

365. (131) **Salwach, C. J. (1981)**, 'Planes, biplanes, and their codes'. *American Mathematical Monthly*, volume **88**, pages 106–125.

[p. 114. Line 9.]

Then $s_i = (\lambda, \ldots, \lambda, r, \lambda, \ldots, \lambda)$, where r inhabits the ith position.

366. (173) **Schaumberger, N. (1989)**, 'Another proof of Chebysheff's inequality'. *The College Mathematics Journal*, volume **20**, pages 141–142.

[p. 141. Title.]

Another Proof of Chebysheff's Inequality

367. (168) **Schwarz, G. E. (1990)**, 'The dark side of the Moebius strip'. *American Mathematical Monthly*, volume **97**, pages 890–897.

[p. 890. Beginning of Section 1.]

What exactly is a Moebius strip? On one hand, it is often defined as the topological space attained by starting with a (closed) rectangle, endowed with the "usual" topology, and identifying two opposite edges point by point with each other, so that each vertex gets identified with the one diagonally across. This "abstract Moebius strip" serves in topology as the canonical example of a nonorientable manifold.

On the other hand, there is a physical model of the abstract strip, and it is usually denoted by the same term ...

368. (160) **Schmid, R. (1992)**, 'Strings, knots, and quantum groups: A glimpse at three 1990 Fields Medalists'. *SIAM Review*, volume **34**, pages 406–425.

[p. 408. Lines 5–4 from bottom.]

As string theory developed, a very rich mathematical structure emerged, but one which now has very little resemblance to strong interaction.

369. (79, 79, 126, 185, 261) **Senechal, M. (1990)**, 'Finding the finite groups of symmetries of the sphere'. *American Mathematical Monthly*, volume **97**, pages 329–335.

[p. 330. Lines 5–6 of Section 3.]

Let H be a finite subgroup of $S0(3)$ of order n. To each element of H (other than the identity) there corresponds an axis that intersects the sphere in two points.

370. (269) **Serrin, J. and D. E. Varberg (1969)**, 'A general chain rule for derivatives and the change of variables formula for the Lebesgue integral'. *American Mathematical Monthly*, volume **76**, pages 514–520.

[p. 515. Lines 4–5.]

All functions considered in this paper are real valued functions of a single real variable.

371. (248) **Shoenfield, J. R. (1967)**, *Mathematical Logic*. Addison-Wes-ley.

[p. 14. Lines 4–1 from bottom.]

We define the *terms* by the generalized inductive definition:
i) a variable is a term; ii) If $\mathbf{u}_1, \ldots, \mathbf{u}_n$ are terms and $\mathbf{f}$ is n-ary, the $\mathbf{f}\mathbf{u}_1, \cdots, \mathbf{u}_n$ is a term.

372. (138, 160) **Shpilrain, V. (1995)**, 'On the rank of an element of a free Lie algebra'. *Proceedings of the American Mathematical Society*, volume **123**, pages 1303–1307.

[p. 1303. Lines 6–5 from bottom.]

If we have an element u of the free Lie algebra L and write $u = u(x_1, \dots, x_n)$, this just means that no generators x_i with $i > n$ are involved in u.

373. (139) **Silverman, J. H. (1992)**, 'Subgroup conditions for groups acting freely on products of spheres'. *Transactions of the American Mathematical Society*, volume **334**, pages 153–181.

[p. 155. Lines 18–16 from bottom.]

The main theorem states that if G is of type I (resp. II), then the larger group $\mathcal{G}$ admits no free, k-cohomologically trivial action on any space $X \sim_2 (S^n)^d$, for any n (resp. for any n not of the form $2^l \cdot 5 - 1$).

374. (200, 219) **Singmaster, D. (1978)**, 'An elementary evaluation of the Catalan numbers'. *American Mathematical Monthly*, volume **85**, pages 366–368.

[p. 366. Line 15–14 from the bottom.]

For example, when $n = 2$, the products above have the Polish form $XXabc$ and $XaXbc$ and the reverse Polish forms $abXcX$ and $abcXX$.

375. (96, 117, 125) **Snyder, W. M. (1982)**, 'Factoring repunits'. *American Mathematical Monthly*, volume **89**, pages 462–466.

[p. 463. Lines 11–14.]

We factor $\Phi_n(b)$ in the ring of algebraic integers of $\mathbb{Q}_n = \mathbb{Q}(\zeta)$. Then

$$\Phi_n(b) = \prod_{\substack{a=1 \\ (a,n)=1}}^{n} (b - \zeta^a) \qquad (1)$$

We now claim that if A is the ideal in R generated by two distinct factors $\zeta - a^{a_1}$ and $\zeta - a^{a_2}$ given in (1), then ...

376. (96, 118, 226) **Sogge, C. D. (1989)**, 'Oscillatory integrals and unique continuation for second order elliptic differential equations'. *Journal of the American Mathematical Society*, volume **2**, pages 491–515.

[p. 494. Lines 2–6.]

... we let $B\,(x,\xi) = \sqrt[m]{P_m\,(x,\xi)}$ and notice that we can factor the symbol of the operator in (1.5) as follows

$$P_m(x,\xi) - \tau^m = (B\,(x,\xi) - \tau) \cdot \left(B^{m-1} + B^{m-2}\tau + \cdots + \tau^{m-1}\right)$$

The second factor is uniformly elliptic in the sense that it is bounded below by a multiple of $(|\xi|^{m-1} + \tau^{m-1})$, while the first factor vanishes for certain $|\xi| \approx \tau$.

377. (123, 204) **Solow, D. (1995)**, *The Keys to Advanced Mathematics: Recurrent Themes in Abstract Reasoning*. BookMasters Distribution Center.

[p. 144. Definition 3.3.]

A set is a **strict subset** of a set B, written $A \subset B$, if and only if $A \subseteq B$ and $A \neq B$.

378. (14, 168, 261) **Solomon, B. (1996)**, 'Tantrices of spherical curves'. *American Mathematical Monthly*, volume **1-3**, pages 30–39.

[p. 31. Last three lines.]

(1.3) Theorem. An immersed circle in $\mathbf{S}^2$ and its tantrix share a regular homotopy class. A tantrix in the equator's class always bounds oriented area $2\pi \pmod{4\pi}$. A trantrix in the other class bounds zero.

379. (118, 118) **Srinivasan, B. (1981)**, 'Characters of finite groups: Some uses and mathematical applications'. *American Mathematical Monthly*, volume **88**, pages 639–646.

[p. 640. Line 7.]

The function $\chi : g \to \mathrm{Trace}\,(\rho(g))$ of G into $\mathbb{C}$ is called the *character* of ρ.

380. (31, 72) **Starke, E. P., editor (1970a)**, 'Problems and solutions'. *American Mathematical Monthly*, volume **77**, pages 765–783.

[p. 774. Problem 5746.]

$$S(a) = \Sigma_{x,y,z} e \left\{ x + y + z + \frac{a}{yz + zx + xy} \right\}$$

381. (72, 107, 232) **Starke, E. P., editor (1970b)**, 'Problems and solutions'. *American Mathematical Monthly*, volume **77**, pages 882–897.

[p. 884. Problem E 2198.]

If $r > 1$ is an integer and x is real, define

$$f(x) = \sum_{k=0}^{\infty} \sum_{j=1}^{r-1} \left[\frac{x + jr^k}{r^{k+1}} \right],$$

where the brackets denote the greatest integer function.

382. (180) **Stangl, W. D. (1996)**, 'Counting squares in $\mathbb{Z}_n$'. *Mathematics Magazine*, volume **69**, pages 285–289.

[p. 285. Lines 1–3.]

An elementary number theory problem is to determine the possible forms of squares among the positive integers. For instance, it is easy to see that any square must be of the form $3k$ or $3k + 1$.

383. (9, 172, 175) **Steen, L. A. (1972)**, 'Conjectures and counterexamples in metrization theory'. *American Mathematical Monthly*, volume **79**, pages 113–132.

[p. 122. Lines 3–6.]

We shall denote by ***WCH*** Jones' hypothesis that $2^{\aleph_0} < 2^{\aleph_1}$, since it is a weak version of *CH*: If $2^{\aleph_0} = \aleph_1$, then $2^{\aleph_0} = \aleph_1 < 2^{\aleph_1}$ by Cantor's theorem. Clearly the consistency of *CH* implies the consistency of *WCH*. The negation of *WCH*, namely $2^{\aleph_0} = 2^{\aleph_1}$, is called the Luzin Hypothesis (***LH***) . . .

384. (35, 51, 73, 77, 93, 125, 222) **Stolarsky, K. B. (1995)**, 'Searching for common generalizations: The case of hyperbolic functions'. *American Mathematical Monthly*, volume **102**, pages 609–619.

[p. 2614. Lines 17–11 from bottom.]

Theorem. *If* $y_1 = y_1(x)$ *and* $y_2 = y_2(x)$ *are functions satisfying* $0 \leq y_1(0) < y_2(0)$ *and the differential equations*

$$\frac{dy_1}{dx} = Cy_2^\alpha, \qquad \frac{dy_2}{dx} = Cy_1^\alpha \tag{4.4}$$

where $C > 0$*, then for some constant* c_0

$$y_2(x) - y_1(x) \to \begin{cases} 0 & \alpha > 0 \\ c_0 & \alpha = 0 \\ \infty & \alpha < 0 \end{cases} \tag{4.5}$$

as x *increases without limit in the (possibly infinite) domain of definition of* $y_1(x)$ *and* $y_2(x)$.

385. (108, 130, 183) **Stolarsky, K. B. (1995)**, 'Searching for common generalizations: The case of hyperbolic functions'. *American Mathematical Monthly*, volume **102**, pages 609–619.

[p. 619. Lines 9–13.]

... then any inequality

$$f(x_1, \ldots, x_n) \geq 0,$$

where the function F is formed by any finite number of rational operations and real exponentiations, is decidably true or false!

386. (239) **Strang, G. (1984)**, 'Duality in the classroom'. *American Mathematical Monthly*, volume **91**, pages 250–254.

[p. 251. Lines 9–11.]

We begin by minimizing not $|x|$ but $\frac{1}{2}|x|^2 = \frac{1}{2}x^T x$, and we introduce the constraint $Ax = b$ through Lagrange multipliers; there are m equations in the constraint, and therefore m multipliers $\lambda_1, \ldots, \lambda_m$.

387. (47, 226) **Strang, G. (1989)**, ‘Patterns in linear algebra’. *American Mathematical Monthly*, volume **96**, pages 105–117.

[p. 107. Lines 15–16.]

Certainly the expression $e^T Ce = c_1 e_1^2 + \cdots + c_4 e_4^2$ is not negative. It is zero only if $e = Ax = 0$.

388. (34, 73, 204, 211) **Straight, H. J. (1993)**, *Combinatorics: An Invitation*. Brooks/Cole. [p. 3. Definition 0.1.1.]

389. (81, 81) **Straight, H. J. (1993)**, *Combinatorics: An Invitation.* Brooks/Cole. [p. 4. Lines 14–15.]

390. (18) **Straight, H. J. (1993)**, *Combinatorics: An Invitation.* Brooks/Cole. [p. 7. Line 9 from the bottom.]

391. (115, 259) **Straight, H. J. (1993)**, *Combinatorics: An Invitation.* Brooks/Cole. [p. 17. 3.]Given sets X and Y, a **function from** X **to** Y is a subset f of $X \times Y$ with the property that, for every $x \in X$, there is a unique $y \in Y$ such that $(x, y) \in f$.

392. (131, 197, 240) **Straight, H. J. (1993)**, *Combinatorics: An Invitation.* Brooks/Cole. [p. 27. Definition 0.2.4.]

393. (38) **Straight, H. J. (1993)**, *Combinatorics: An Invitation.* Brooks/Cole. [p. 119. Line 7 from bottom.]

394. (96, 127) **Suryanarayana, D. (1977)**, 'On a class of sequences of integers'. *American Mathematical Monthly*, volume **84**, pages 728–730.

[p. 728. Lines 14–15.]

Let $\{a_n\}$ be an increasing sequence of positive integers such that $\log a_n / q_n \log q_n \to 0$ as $n \to \infty$, where q_n is the least prime factor of n.

395. (117, 117, 118) **Steve Surace, J. (1990)**, 'The Schrödinger equation with a quasi-periodic potential'. *Transactions of the American Mathematical Society*, volume **320**, pages 321–370.

[p. 321. Abstract.]

We consider the Schrödinger equation

$$-\frac{d^2}{dx^2}\psi + \epsilon\left(\cos x + \cos\left(\alpha x + \vartheta\right)\right)\psi = E\psi$$

where ...

396. (47, 107, 135, 269) **Swartz, C. and B. S. Thomson (1088)**, 'More on the Fundamental Theorem of Calculus'. *American Mathematical Monthly*, volume **95**, pages 644–648.

[p. 644. Lines 14–8.]

It is this requirement of being able to uniformly partition the interval that limits the scope of the Riemann integral. It would be much more desirable to somehow allow"variable length" partitions; for example, if one were attempting to approximate the area under the graph of $f(x) = 1/\sqrt{x}$, $0 < x \leq 1$, it would be natural to take the subintervals in an approximating partition to be very fine near the singularity $x = 0$.

397. (118, 118) **Talagrand, M. (1986)**, 'Derivations, L^{Ψ}-bounded martingales and covering conditions'. *Transactions of the American Mathematical Society*, volume **293**, pages 257–291.

[p. 257. Abstract.]

Let (Ω, Σ, P) be a complete probability space. Let $(\Sigma_j)_{j \in J}$ be a directed family of sub-σ-algebras of Σ. Let (Φ, Ψ) be a pair of conjugate Young functions.

398. (52, 59, 137, 222) **Talagrand, M. (1990)**, 'The three-space problem for L^1'. *Journal of the American Mathematical Society*, volume **3**, pages 9–29.

[p. 9. Lines 9–8 from bottom.]

For simplicity, let us say that a Banach space *contains a copy of* L^1 if it contains a subspace isomorphic to L^1.

399. (8, 122) **Teitelbaum, J. T. (1991)**, 'The Poisson kernel for Drinfeld modular curves'. *Journal of the American Mathematical Society*, volume **4**, pages 491–511.

[p. 494. Lines 1–4.]

... may find a homeomorphism $x : E \to \mathbb{P}^1_k$ such that

$$x(\gamma u) = \frac{ax(u) + b}{cx(u) + d}.$$

We will tend to abuse notation and identify E with $\mathbb{P}^1_k$ by means of the function x.

400. (21) **Temple, B. and C. A. Tracy (1992)**, 'From Newton to Einstein'. *American Mathematical Monthly*, volume **99**, pages 507–521.

[p. 507. Lines 3–6.]

In particular, he derived Kepler's laws of motion from the assumption that the sun *pulls* on a planet with a *force* that varies inversely with the square of the distance from the sun to the planet.

401. (198) **Temple, B. and C. A. Tracy (1992)**, 'From Newton to Einstein'. *American Mathematical Monthly*, volume **99**, pages 507–521.

[p. 518. Line before (4.11).]

Plugging in we obtain:

402. (232) **Tews, M. C. (1970)**, 'A continuous almost periodic function has every chord'. *American Mathematical Monthly*, volume **77**, pages 729–731.

[p. 730. Third line from the bottom.]

$$\left|\sin\left[\frac{2\pi}{a}\left(\frac{a}{4}+t\right)\right]-\sin\left(\frac{2\pi}{a}\cdot\frac{a}{4}\right)\right| < \sin\frac{2\pi}{a}p$$

403. (181, 234, 240) **Thielman, H. P. (1953)**, 'On the definition of functions'. *American Mathematical Monthly*, volume **60**, pages 259–262.

[p. 260. Lines 16–14 from bottom.]

A function f whose domain of definition is X, and whose range is Y is frequently denoted by $f : X \to Y$, and is referred to as a function *on* X *onto* Y.

404. (99) **Thurston, H. (1993)**, 'On tangents'. *Mathematics Magazine*, volume **66**, pages 227–235.

[p. 228. Lines 1–4 after the figures.]

Eventually a good definition of tangent was devised. Succinctly put,

the tangent is the limit of the secant.

Let us formalize this definition. I shall give a formal definition, not of a tangent to a curve, but of a tangent to a set of points.

405. (189, 239, 250, 276) **Tits, J. (1964)**, 'Algebraic and abstract simple groups'. *The Annals of Mathematics, 2nd Ser.*, volume **80**, pages 313–329.

[p. 321. Lines 25–29.]

... (the symbols $A_2, A_3, \cdots, G_2$ have their usual meaning, and the left superscript denotes the degree of $\tilde{k}/k$ when $\tilde{k} \neq k$, i.e, when G does not split over k)

if $k = F_2$, groups of type A_2, ${}^2A_3$, ${}^2A_4$, B_3 and ${}^3D_4$;

if $k = F_3$, groups of type A_2, ${}^2A_2$, ${}^2A_3$, B_2 and ${}^3D_4$ and G_2.

406. (160) **Trangenstein, J. A. and J. B. Bell (1989)**, 'Mathematical structure of the black-oil model for petroleum reservoir simulation'. *SIAM Journal on Applied Mathematics*, volume **49**, pages 749–783.

[p. 749. Line 9.]

In this paper we analyze the mathematical structure of the black-oil flow equations.

407. (242, 266) **Trowbridge, D. (1875)**, 'On the maxima and minima of algebraic polynomials'. *The Analyst*, volume **2**, pages 1–4.

[p. 1. Lines 1–2.]

Let $u = A_1x^n + A_2x^{n-1} + A_3x^{n-2} + \ldots + A_{n-1} = \psi x \ldots$ The expression ψx is read "function of x".

408. (125, 264) **Underwood, R. S. (1954)**, 'Extended analytic geometry as applied to simultaneous equations'. *American Mathematical Monthly*, volume **61**, pages 525–542.

[p. 525. Lines 17–20.]

The example also illustrates the point that it is not always easy to decide whether two equations in more than two unknowns are consistent, quite aside from the matter of producing in that case a real solution.

409. (34, 139) **van Lint, J. H. and R. M. Wilson (1992)**, *A Course in Combinatorics.* Cambridge University Press.

[p. 35. Lines 8–4 from bottom.]

... We shall show that a larger matching exists. (We mean larger in *cardinality*; we may not be able to find a complete matching containing these particular m edges.)

410. (15, 47, 198, 261) **Van Douwen, E. K., D. J. Lutzer, and T. C. Przymusiński (1977)**, 'Some extensions of the Tietze-Urysohn Theorem'. *American Mathematical Monthly*, volume **84**, pages 435.

[p. 435. Theorem A.]

If A is a closed subspace of the normal space X then there is a function $\eta : C^*(A) \to C^*(A)$ such that for every $f \in C^*(A)$, $\eta(F)$ extends F and has the same bounds as F.

411. (168) **Vaught, R. L. (1973)**, 'Some aspects of the theory of models'. *American Mathematical Monthly*, volume **80**, pages 3–37.

[p. 3. Lines 6–10.]

For example, each of the properties of being a group, an Abelian group, or a torsion-free Abelian group os expressible in the so-called elementary language (or first-order predicate calculus). Thus, instead of saying that the group $\mathcal{G}$ is Abelian, we can sat it is a model of the elementary sentence $\forall x \forall y (x \circ y = y \circ x)$. Such properties are also called elementary.

412. (56, 101, 210) **Vaught, R. L. (1973)**, 'Some aspects of the theory of models'. *American Mathematical Monthly*, volume **80**, pages 3–37.

[p. 11. Lines 19–22.]

... the existence of such an $\mathfrak{A}$ is of basic importance in the remarkable work of Gödel and Cohen on the consistency and independence of the continuum hypothesis and other basic propositions in set theory.

413. (57, 276) **Vaught, R. L. (1973)**, 'Some aspects of the theory of models'. *American Mathematical Monthly*, volume **80**, pages 3–37.

[p. 25. Lines 12–11, 6–2 from bottom.]

T is called ω-**complete** if whenever $T \vdash \forall v_0 (\theta_n (v_0) \to \phi (v_0))$ for all n, then $T \vdash \forall v_0 \phi (v_0)$ ... It is worthwhile noting that in contrapositive form the definition of ω-complete reads:

(3) T is ω-complete if and only if for any 1-formula ϕ, if $T + \exists v_0 \phi (v_0)$ has a model, then for some n, $T \exists v_0 (\theta_n (v_0) \wedge \phi (v_0))$ has a model.

414. (32, 171) **Verner, J. H. (1991)**, ‘Some Runge-Kutta formula pairs’. *SIAM Journal on Numerical Analysis*, volume **28**, pages 496–511.

[p. 501. Lines 1–2 under formula (21″).]

This may be written as

$$\sum_{j=4}^{7}\left(\sum_i b_i a_{ij}\right)\cdot\left(\sum_k a_{jk}c_k^q - \frac{c_j^{q+1}}{q+1}\right) = 0$$

by invoking (15) to imply that the first bracket is zero for $j = 2, 3$. Since the second bracket is zero for $4 \leq j \leq 6$ by (17″), and …

415. (32) **Wallach, N. R. (1993)**, 'Invariant differential operators on a reductive Lie algebra and Weyl group representations'. *Journal of the American Mathematical Society*, volume **6**, pages 779–816.

[p. 786. Lines 8–7 from bottom.]

If $f, g \in \mathcal{P}(V_0 \times V_0^*)$ then let $\{f, g\}$ (the Poisson bracket of f and g) be as in Appendix 1.

416. (28, 229) **Waterhouse, W. C. (1994)**, 'A counterexample for Germain'. *American Mathematical Monthly*, volume **101**, pages 140–150.

[p. 141. Lines 9–6 from bottom.]

Suppose n is an odd prime. In modern terms, Gauss has shown that the field generated over the rationals by the n-th roots of unity contains $\sqrt{\pm n}$; here we must take the plus sign when n is of the form $4k+1$ and the minus sign when n is of the form $4k+3$.

417. (250) **Weiss, G. (1970)**, ‘Complex methods in harmonic analysis’. *American Mathematical Monthly*, volume **77**, pages 465–474.

[p. 465. Lines 1–10.]

First, we plan to show how properties of analytic functions of a complex variable can be used to obtain several results of classical harmonic analysis (that is, the theory of Fourier series and integrals of one real variable). This will be done in Section 1. Second, in Section 2 we shall indicate how some of these applications of the theory of functions can be extended to Fourier analysis of functions of several variables.

Some of these applications of the theory of functions seem very startling since the results obtained appear to involve only the theory of functions of a *real* variable or the theory of measure.

418. (147) **Whittlesey, E. F. (1960)**, 'Finite surfaces a study of finite 2-complexes'. *Mathematics Magazine*, volume **34**, pages 11–22.

[p. 15. Lines 1–7.]

To determine the global structure of a complex we need to have first a knowledge of the local structure, of behaviour in a neighborhood of a point; with this understanding of local structure, we can solve the problem of recognition of a 2-complex by putting the pieces together, as it were. In the case of graphs, the problem of local structure is completely resolved by a knowledge of the degree of a vertex. The matter is less simple for surfaces.

419. (250) **Wilson, E. B. (1918)**, 'The mathematics of aëerodynamics'. *American Mathematical Monthly*, volume **25**, pages 292–297.

[p. 295. Lines 22–25.]

Now by the method of "conformal representation" of the theory of functions the pressure exerted on a plane wing of various shapes, by the motion of the air, may in some cases be calculated, and the center of pressure may also be found.

420. (31, 131, 135) **Wilf, H. S. (1985)**, 'Some examples of combinatorial averaging'. *American Mathematical Monthly*, volume **92**, pages 250–261.

[p. 253. Lines 14–16.]

Therefore, if $n > 1$ we have

$$Q(n) = \frac{1}{n!}\left\{\sum_{i=1}^{n}(n-1)! + \sum_{\substack{i,j=1 \\ i\neq j}}^{n}(n-2)!\right\}$$

What sort of creature now inhabits the curly braces?

421. (51, 130, 251) **Williams, H. P. (1986)**, ‘Fourier’s method of linear programming and its dual’. *American Mathematical Monthly*, volume **93**, pages 681–695.

[p. 682. Lines 9–5 from bottom.]

Constraint C0 is really a way of saying we wish to maximise z where

$$z \leq -4x_1 + 5x_2 + 3x.$$

By maximising z we will “drive” it up to the maximum value of the objective function. It would clearly be possible to treat C0 as an equation but for simplicity of exposition we are treating all constraints as $\leq$ inequalities.

422. (116) **Wilf, H. S. (1989)**, 'The editor's corner: The white screen problem'. *American Mathematical Monthly*, volume **96**, pages 704–707.

[p. 704. Lines 11–8 from bottom.]

To translate the question into more precise mathematical language, we consider a grid of MN lattice points

$$G = \{(i, j) \mid 0 \leq i \leq M - 1; 0 \leq j \leq N - 1\}$$

and we regard them as being the vertices of a graph.

423. (47, 48, 108, 118, 209) **Witte, D. (1990)**, 'Topological equivalence of foliations of homogeneous spaces'. *Transactions of the American Mathematical Society*, volume **317**, pages 143–166.

[p. 144. Lines 16–13 from bottom.]

By composing $\tilde{f}$ with the inverse of σ, we may assume the restriction of $\tilde{f}$ to $\mathbf{Z}^n$ is the identity. Because

(**)$\mathbf{R}/\mathbf{Z}$ is compact,

this implies that $\tilde{f}$ moves points by a bounded amount ...

424. (14) **Wolenski, P. R. (1993)**, 'The semigroup property of value functions in Lagrange problems'. *Transactions of the American Mathematical Society*, volume **335**, pages 131–154.

[p. 146. Lines 2–3.]

The right hand side of (5.36) equals $\frac{2\epsilon}{\lambda}(1 + h\lambda)^k - 1)$ [sic], which is always less than or equal to $\frac{2\epsilon}{\lambda}e^{\lambda T}$.

425. (267) **Wulbert, D. E. (1978)**, 'The rational approximation of real functions'. *American Journal of Mathematics*, volume **100**, pages 1281–1315.

[p. 1282. Lines 15–17 and 23–25.]

A complex function η defined on $[0, 1]$ still has exactly one best approximation from $\mathcal{P}_n(\mathbb{C})$. ... If the function being approximated is the real function f, then its best approximation in $\mathcal{P}_n(\mathbb{C})$ is also real ...

426. (201, 203) **Yetter, D. N. (1990)**, 'Quantales and (noncommutative) linear logic'. *Journal of Symbolic Logic*, volume **55**, pages 41–64.

[p. 44. Lines 14–13 from bottom.]

In our formalism we adopt prefix notation in preference to the infix/postfix notation used by Girard ...

427. (193) **Yu, H. B. (1998)**, 'On the Diophantine equation $(x+1)^y - x^z = 1$'. *American Mathematical Monthly*, volume **105**, pages 656–657.

[p. 656. Last line.]

$$\big((x+1)^{y_1} - 1\big)\big((x+1)^{y_1}) + 1\big) = x^z$$

428. (32, 47, 181) **Zabell, S. L. (1995)**, 'Alan Turing and the Central Limit Theorem'. *American Mathematical Monthly*, volume **102**, pages 483–494.

[p. 486. Lines 4–6.]

Feller (1937) showed that if normal convergence occurs (that is, condition (2.2) holds), but condition (2.4) also obtains, then

$$\frac{1}{\rho}\frac{X_{m_k}}{s_{n_k}} \Rightarrow (0,1)\,.$$

429. (19, 106) **Zalcman, L. (1975)**, 'A heuristic principle in complex function theory'. *American Mathematical Monthly*, volume **82**, pages 813–818.

[p. 813. Lines 4–3 from the bottom.]

Here $f^\sharp$ is the *spherical derivative* of the function; the present notation … is better adapted for displaying the argument of the function explicitly.

430. (126, 131, 134, 143) **Zalcman, L. (1980)**, 'Offbeat integral geometry'. *American Mathematical Monthly*, volume **87**, pages 161–175.

[p. 162. Lines 6–10.]

Accordingly, let

$$\hat{f}(\xi, \eta) = \int \int f(x, y)e^{i(\xi x + \eta y)} dx\, dy$$

be the Fourier transform of f. We shall need only two facts about $\hat{f}$: it is continuous, and the correspondence between f and $\hat{f}$ is one-to-one. In particular, if $\hat{f} = 0$ then $f = 0$ (almost everywhere).

431. (118) **Zander, V. (1972)**, 'Fubini theorems for Orlicz spaces of Lebesgue-Bochner measurable functions'. *Proceedings of the American Mathematical Society*, volume **32**, pages 102–110.

[p. 102. Lines 1–2 of Abstract.]

Let (X, Y, υ) be the volume space formed as the product of the volume spaces (X_i, Y_i, υ_i) $(i = 1, 2)$.

432. (194, 239) **Zulli, L. (1996)**, 'Charting the 3-sphere—an exposition for undergraduates'. *American Mathematical Monthly*, volume **103**, pages 221–229.

[p. 227. Beginning of Section 5.]

Let us return for a moment to the circle $S^1 \subseteq \mathbf{C} = \mathbf{R}^2$.

Missing Links

mental representation – conceptual blending
elementary – informal jargon
well-defined – mod
name – names from other languages
pronunciation – names from other languages