Hydrodynamic analysis of porous spheres with infiltrated peripheral shells in linear flow fields

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Abstract

The velocity fields inside and around porous spheres with infiltrated peripheral shells were solved for three different far-field flows: simple shear, planar elongation and uniaxial extension. The flow was considered to obey Stokes’ law around the porous sphere and Brinkman’s extension of Darcy’s law within an infiltrated spherical shell beneath the sphere surface. The infiltrated layer, resulting from capillary action, was assumed to have a thickness that remained unaffected when the porous sphere was subjected to external flow fields. The cumulative hydrodynamic force exerted by the fluid upon the solid portion of the porous sphere was calculated for spherical caps with planar fracture surfaces. The magnitude of the tensile component of the hydrodynamic force per unit area of the base of the cap was shown to increase with the size of the cap and with the thickness of the infiltrated layer. On the other hand, the magnitude of the shear component of the hydrodynamic force per unit area of the base of the cap exhibited a maximum for a given cap size. The cap size that maximized the shear force increased with the thickness of the infiltrated layer. However, in contrast to the tensile force, for a constant cap size, the shear force did not necessarily increase with the thickness of the infiltrated layer. The hydrodynamic force exerted by the fluid upon the solid was found to increase with the inverse of the permeability of the porous sphere. The limit for very low permeability was compared to the case of an impermeable sphere. In the case of the tensile force, there was a difference between the low permeability limit for a permeable sphere and the value corresponding to an impermeable sphere. This difference could be attributed to the presence of fluid within the permeable sphere. The fluid contained in the permeable sphere could transmit pressure to the solid, a contribution absent in the case of an impermeable sphere.

Keywords: Hydrodynamics; Linear flow fields; Solid mechanics; Porous media; Permeable sphere; Mathematical modelling

1. Introduction

The purpose of this study is to analyze the stress distribution inside a partially infiltrated porous sphere of uniform permeability $k$, in various linear flow fields (simple shear, planar elongation and uniaxial extension). The permeable sphere of radius $a$ is characterized by an infiltrated outer shell of constant thickness $d$ surrounding a dry core of radius $R$, as depicted in Fig. 1.

Results of this analysis can help understand hydrodynamic dispersion processes of fine-particle clusters (agglomerates) in sheared fluids. Hydrodynamic dispersion of agglomerates occurs when the action of fluid stresses produces internal stresses that exceed the cohesive strength of the agglomerate. The dispersion of agglomerates depends therefore strongly on the geometry and strength of the flow field and on the cohesive strength of the agglomerates to be dispersed. Additionally, immersion of permeable agglomerates in their processing media results in their progressive infiltration by the fluid. This phenomenon was observed and monitored earlier in the case of silica, calcium carbonate, carbon black and titanium dioxide agglomerates (Bohin, Zloczower, & Feke, 1994; Levresse, Zloczower, Feke, Bomal, & Bortzmeyer, 1999; Yamada, Zloczower, & Feke, 1998; Bohin, Feke, & Zloczower, 1995). The mechanics of stress transmission from fluid to agglomerate is influenced by the amount of flow through the infiltrated portion of the agglomerate. The presence of fluid within agglomerate can also modify its cohesive strength. Consequently, matrix infiltration affects the dispersion propensity of...

The partially infiltrated porous sphere depicted in Fig. 1 is a good model for typical agglomerates. Indeed the symmetric infiltration of liquids into spherical agglomerates was observed experimentally in the case of silica using phase contrast microscopy (Bohin et al., 1994). The kinetics of infiltration was shown to obey a theoretical model based on Darcy’s law (Bohin et al., 1994; Levresse et al., 1999; Yamada et al., 1998):

\[
2a^3 - 3a^2 + 1 = r_t, \tag{1}
\]

where the rate of infiltration \( r_t \) depends on characteristics of the agglomerate (morphology and packing density) and of the fluid (viscosity, surface tension, contact angle). The inverse of \( r_t \) represents the time necessary for the fluid to completely fill the agglomerate \( (R = 0) \) which typically ranges from one to several hours (Bohin et al., 1994, 1995; Levresse et al., 1999; Yamada et al., 1998). Since the hydrodynamic relaxation time is much shorter than the characteristic infiltration time, a pseudo-steady-state analysis in which the thickness of the infiltrated layer remains constant (the approach in this paper) is appropriate.

In this work, we also considered the case of fully filled porous spheres \( (R = 0) \) with various permeabilities. For spheres with very limited permeability \( (k \rightarrow 0) \), the hydrodynamic force is expected to be different from that acting on an impermeable sphere due to the transmission of pressure by the fluid contained in the porous sphere.

2. Modeling approach

2.1. Governing equations

The fluid is assumed to be incompressible and Newtonian with viscosity \( \mu \). At every step of the calculation the more appropriate of the Cartesian or spherical coordinate systems is used (Fig. 2).

The flow around the sphere obeys the Stokes and continuity equations:

\[
\mu \nabla^2 \mathbf{u}^e = \nabla \mathbf{p}^e, \quad r \geqslant a, \tag{2}
\]

\[
\nabla \cdot \mathbf{u}^e = 0, \quad r \geqslant a, \tag{3}
\]

while the flow within the infiltrated shell of the porous sphere is accounted for by the continuity equation and Brinkman’s extension of Darcy’s law, which is known to be well adapted to describe flow through porous media (Neale, Epstein, & Nader, 1973; Adler & Mills, 1979; Masliyah, Neale, Malyea, & Van De Ven, 1987; Sonntag & Russel, 1987; Davis & Stone, 1993; Padmavathi, Amaranath, & Nigam, 1993):

\[
\mu \nabla^2 \mathbf{u}^i - \frac{\mu}{k} (\mathbf{u}^i - \mathbf{u}_s) = \nabla \mathbf{p}^i, \quad R \leqslant r \leqslant a, \tag{4}
\]

\[
\nabla \cdot \mathbf{u}^i = 0, \quad R \leqslant r \leqslant a. \tag{5}
\]

The superscript \( e \) refers to the flow around the sphere while the superscript \( i \) refers to the flow within the infiltrated shell of the porous sphere. At any given point, a quantity marked with the superscript \( i \) represents an average over a macroscopic volume around this point and cannot be considered as being the true value of the quantity at the microscopic level. The velocity term \( \mathbf{u}_s \) corresponds to the overall motion of the sphere (for instance, in the case of simple flow, the sphere rotates due to the vorticity of the flow field).

2.2. Boundary conditions

Far from the sphere \( (r = \infty) \) the velocity of the fluid is equal to the undisturbed velocity \( \mathbf{u}_0 \) (velocity field in the absence of the sphere) characteristic of the flow field under consideration (simple shear, planar elongation or uniaxial extension).
Across the porous sphere surface \((r = a)\) velocity vectors and total stresses are continuous:

\[
\mathbf{u}'(a, \theta, \phi) = \mathbf{u}''(a, \theta, \phi),
\]

\[
\mathbf{n} \cdot \mathbf{T}'(a, \theta, \phi) = \mathbf{n} \cdot \mathbf{T}''(a, \theta, \phi)
\]

in which \(\mathbf{n}\) is the radial unit vector \((\mathbf{n} = r/r)\).

At the interface between the dry and the wet region \((r = R)\) the radial velocity is zero (no flow-induced infiltration),

\[
u'_r(R, \theta, \phi) = 0,
\]

and it is assumed that no tangential stress is imparted on the fluid:

\[
T'_r(R, \theta, \phi) = 0,
\]

\[
T'_\theta(R, \theta, \phi) = 0.
\]

Indeed, the outer surface of the dry core can be viewed as a composite surface, consisting of solid and gas (air). For highly porous spheres (such as typical agglomerates), where the dry core surface is predominantly composed of air, a zero-tangential stress boundary condition is believed to be more appropriate than a no-slip boundary condition.

The total stress tensors for both the flow around and within the agglomerate can be calculated from the respective velocity fields. Since the fluid is incompressible and Newtonian,

\[
\mathbf{T} = -p \mathbf{I} + \mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T),
\]

where \(\mathbf{I}\) is the identity tensor.

### 2.3. Solution

Padmavathi et al. (1993) and Raja Sekhar, Padmavathi, and Amarath (1997) provided a complete general solution of Brinkman’s and Stokes’ equations using scalar fields. The scalar functions \(A_0\) and \(B_0\) associated with the undisturbed flow field are chosen so as to obey the following equation:

\[
\mathbf{u}_0 = \nabla \times \nabla \times (rA_0) + \nabla \times (rB_0),
\]

\(A_0\) and \(B_0\) are solutions of

\[
\nabla^4 A_0 = 0, \quad \nabla^2 B_0 = 0.
\]

The velocity and pressure fields around the sphere \((r \geq a)\) are expressed as follows:

\[
\mathbf{u}' = \nabla \times \nabla \times (rA') + \nabla \times (rB'),
\]

\[p' = p_0 + \mu \frac{\partial}{\partial r} [r \nabla^2 A'],\]

in which

\[
\nabla^4 A' = 0, \quad \nabla^2 B' = 0,
\]

\[
\lim_{r \to \infty} A' = A_0, \quad \lim_{r \to \infty} B' = B_0,
\]

while the velocity and pressure fields within the infiltrated region of the porous sphere \((R \leq r \leq a)\) obey:

\[
\mathbf{u}' = \nabla \times \nabla \times (rA') + \nabla \times (rB') + \mathbf{u}_0,
\]

\[
p' = p_0 + \mu \frac{\partial}{\partial r} [r (\nabla^2 - \lambda^2) A'], \quad \lambda^2 = \frac{1}{k}.
\]

\[
A' = A'_1 + A'_2,
\]

\[
\nabla^2 A'_1 = 0, \quad (\nabla^2 - \lambda^2) A'_2 = 0, \quad (\nabla^2 - \lambda^2) B'_0 = 0.
\]

For the sake of simplicity, all lengths are made dimensionless on the square root of the porous sphere permeability:

\[
\tilde{r} = \frac{r}{\sqrt{k}}, \quad \tilde{z} = \frac{z}{\sqrt{k}}, \quad \tilde{\beta} = \frac{R}{\sqrt{k}}, \quad \delta = \frac{d}{\sqrt{k}}.
\]

The specific solutions for each of the three flow fields are detailed in the following. The equations shown exclusively apply to partially filled porous spheres \((R \neq 0)\). The case of fully filled porous spheres \((R = 0)\) is treated separately in a subsequent section of this paper.

#### 2.3.1. Simple shear

The undisturbed simple shear flow field can be represented by the following equations:

\[
u_{0,x} = \dot{e} y, \quad u_{0,y} = u_{0,z} = 0,
\]

where \(\dot{e}\) is the magnitude of the rate of deformation tensor.

Following is a set of scalar functions \(A_0\) and \(B_0\) obeying Eq. (12):

\[
A_0 = \frac{\dot{e}}{6} xy = \frac{\dot{e} k}{6} \tilde{z} \sin \theta \cos \phi \sin \phi,
\]

\[
B_0 = -\frac{\dot{e}}{2} z = -\dot{e} \sqrt{k} \tilde{z} \sin \theta \cos \phi.
\]

The velocity and pressure fields around the porous sphere and within its infiltrated shell can be written using the following scalar functions:

\[
A' = \frac{\tilde{z}^2}{6} + I \frac{\tilde{z}}{\xi} + J \frac{f(0, \phi)}{\xi},
\]

\[
B' = -\frac{\tilde{z}}{2} \frac{K}{\xi} g(0, \phi).
\]
\[ A_1' = \left( C \frac{\xi^2}{3} + \frac{D}{\xi^3} \right) f(\theta, \phi), \]  
\[ A_2' = \left\{ \begin{array}{l} E \left[ \frac{\sinh \xi}{\xi^3} + \frac{1}{\xi^3} \sin \xi - \frac{3}{\xi^2} \cosh \xi \right] f(\theta, \phi), \\ + F \left[ \frac{\cosh \xi}{\xi^3} + \frac{1}{\xi^3} \cos \xi - \frac{3}{\xi^2} \sinh \xi \right] f(\theta, \phi), \end{array} \right. \]  
\[ B' = \left[ G \left( - \frac{\sinh \xi}{\xi^3} + \frac{\cosh \xi}{\xi^3} \right) \right] g(\theta, \phi) \]  
with \( f(\theta, \phi) = \frac{k}{\xi} \sin \theta \cos \theta \sin \phi \) and \( g(\theta, \phi) = \xi \sqrt{k} \sin \theta \cos \phi \).

In the case of simple shear flow field, the sphere is rotating about the \( z \)-axis with a constant angular velocity \( \omega \):
\[
u_{z,\phi} = 0, \quad u_{z,\theta} = \omega \sqrt{k} \xi \sin \phi, \]  
\[
u_{z,\phi} = \omega \sqrt{k} \xi \cos \theta \cos \phi. \]  
In the important case of a torque free sphere (steady state for a sphere left to itself), the condition of zero torque at the sphere surface writes:
\[
\int_{0}^{2\pi} \int_{0}^{\pi} M_{z,r=a} a^2 \sin \theta \, d\theta \, d\phi = 0, \quad \mathbf{M} = \mathbf{r} \times (\mathbf{n} \cdot \mathbf{T}). \]  
Along with boundary conditions 6–10, Eq. (32) allows the determination of the values of the parameters \( C, D, E, F, G, H, I, J, K \) and \( \omega \). The angular velocity \( \omega \) of the torque free sphere is found to be \( \hat{\omega} / 2 \). This value corresponds to the vorticity of the fluid (Adler & Mills, 1979). The three parameters \( G, H, K \) are zero in this case. However, this does not hold true if the sphere is held immobile or rotates with an externally imposed angular velocity, in which cases \( G, H, K \) and \( \omega \) are different from zero (cases not shown in this paper). The other parameters \( C, D, E, F, I, J \) and \( J \) are reported in Appendix A and are valid irrespective of the rotation condition.

### 2.3.2 Planar elongation

The planar elongation flow field can be represented by the following equations:
\[
u_{0,z} = -\hat{v} z, \quad u_{0,y} = \hat{v} y, \quad u_{0,x} = 0. \]  
The following scalar functions \( A_0 \) and \( B_0 \) satisfy Eq. (12):
\[
A_0 = \frac{\hat{v}}{6} \left( y^2 - x^2 \right) = \frac{\hat{v} \xi^2}{6} \left( \cos^2 \theta - \sin^2 \theta \sin^2 \phi \right), \]  
\[
B_0 = 0. \]  
The solution of the problem can be written with the scalar functions given in Eqs. (26)–(30) for the case of simple shear flow field but with \( f(\theta, \phi) = \hat{v} k (\cos^2 \theta - \sin^2 \theta \sin^2 \phi) \) and \( g(\theta, \phi) = 0 \).

The porous sphere (assumed to be positioned at the origin of the coordinate system) is stationary (\( u_r = 0 \)). The values of the parameters \( C, D, E, F, I \) and \( J \) are similar to those corresponding to the case of simple shear flow and can be found in Appendix A.

#### 2.3.3 Uniaxial extension

The uniaxial extension flow field can be represented by the following equations:
\[
u_{0,x} = -\frac{\hat{v}}{2} x, \quad u_{0,y} = \hat{v} y, \quad u_{0,z} = -\frac{\hat{v}}{2} z. \]  
The following scalar functions \( A_0 \) and \( B_0 \) are solution of Eq. (12):
\[
A_0 = \frac{\hat{v}}{6} \left( y^2 - x^2 - \frac{z^2}{2} \right) = \frac{\hat{v} \xi^2}{6} \left( 1 - \frac{3}{2} \sin^2 \theta \right), \]  
\[
B_0 = 0. \]  
The solution of the problem can be written with the scalar functions given in Eqs. (26)–(30) for the case of simple shear flow field but with \( f(\theta, \phi) = \hat{v} k (1 - 1.5 \sin^2 \theta) \) and \( g(\theta, \phi) = 0 \).

As in the planar elongation case, the porous sphere (if positioned at the center of the coordinate system) is stationary (\( u_r = 0 \)). The values of the parameters \( C, D, E, F, I \) and \( J \) are identical to those corresponding to the two previous flow fields and are given in Appendix A.

### 2.4 Solid mechanics

The cumulative hydrodynamic force exerted by the fluid upon a volume \( V \) of solid is calculated by integrating the stresses in the fluid:
\[
\mathbf{F} = \iint_S (\mathbf{n} \cdot \mathbf{T}) \, dS, \]  
in which \( S \) is the surface surrounding the wet portion of the volume \( V \) (within the volume \( V \), regions which contain no fluid do not contribute to the integral).

In the following the integration volumes under consideration are spherical caps with planar bases. As shown in Fig. 3, the position of a cap is given by the angular coordinates \( \Theta \) and \( \Phi \) of the unit vector \( \mathbf{N} \) passing through the center of the cap. The size of the spherical cap can be characterized by the height \( h \) (or \( \eta \) when made dimensionless on the square root of the porous sphere permeability, as in Eq. (22)). As represented in Fig. 3, the surface of the spherical cap can be divided into the surface \( S_1 \) at the base of the cap and the outer surface \( S_2 \), part of the sphere surface.
We choose the surface of the base of the cap $S_1$ to calculate the hydrodynamic stresses acting on the fragment (spherical cap). Then the force per unit area of the base of the cap is given by

$$\Sigma = \frac{\mathbf{F}}{\iint_{S_1} dS}.$$  \hfill (40)

As shown in Fig. 3, the force $\mathbf{F}$ can be decomposed into a tensile component parallel to $\mathbf{N}$ and a shear component perpendicular to $\mathbf{N}$:

$$\mathbf{F} = \mathbf{F}_{\text{tensile}} + \mathbf{F}_{\text{shear}} \quad \text{with} \quad \mathbf{F}_{\text{tensile}} = F_N \mathbf{N}$$

and

$$\mathbf{F}_{\text{shear}} = F_\phi \mathbf{e}_\phi + F_\Theta \mathbf{e}_\Theta. \quad \text{(41)}$$

### 2.5. Case of a fully filled porous sphere ($R = 0$)

The solution for the case $R = 0$ can be found in the literature (Adler & Mills, 1979; Padmavathi et al., 1993). For the three flow fields, the analytical solutions for the case $R = 0$ are included in our general solution (Eqs. (26)–(30)) with the simplification that $D$, $F$ and $H$ are set equal to zero. The remaining parameters are adjusted so as to satisfy a reduced system of boundary conditions (Eqs. (6) and (7)), i.e. only those pertaining to the sphere surface.

For the simple shear flow field, $G$ and $K$ are zero and $\omega$ equals $\pi/2$ in the case of a torque free sphere. The parameters $C$, $E$, $I$ and $J$ for the solution of the problem are reported in Appendix B and apply to the three flow fields. Similarly to the case of partially filled porous spheres ($R \neq 0$), the cumulative hydrodynamic force acting on solid fragments is calculated using Eq. (39).

### 2.6. Case of an impermeable sphere

For simple shear, uniaxial extension and planar elongation, the solutions for the flow fields surrounding an impermeable sphere are classical and can be easily retrieved from the equations given earlier in this paper, using solely the quantities pertaining to the flow around the porous sphere $A^*$ and $B^*$.

For the simple shear flow field, $K$ is zero and $\omega$ is equal to $\pi/2$ in the case of a torque free sphere. For all three flow fields, a no-slip boundary condition at the sphere surface is evoked to determine the unknown parameters $I$ and $J$ which are reported in Appendix C:

$$\mathbf{u}(a,\Theta,\Phi) = \mathbf{u}_s(a,\Theta,\Phi). \quad \text{(42)}$$

The cumulative hydrodynamic force transmitted by the fluid to a spherical cap on the sphere is simply the integration of the stresses in the fluid over the surface $S_2$:

$$\mathbf{F} = \int_{S_2} (\mathbf{n} \cdot \mathbf{T}) dS. \quad \text{(43)}$$

### 3. Results and discussion

The hydrodynamic force transmitted by the fluid to the solid portion of the porous sphere is affected both by characteristics of the cap (size and angular orientation) and properties of the porous sphere (permeability, thickness of the infiltrated layer).

The cumulative hydrodynamic force is calculated in this paper for spherical caps of different sizes, for porous spheres with various infiltration levels, in the three different flow fields, using Eq. (39). The force per unit area of the base of the cap is obtained using Eq. (40). For the simple shear and planar elongation flow geometries the magnitude of the force is maximum for caps for which $\Phi = 90^\circ$ (or $270^\circ$). Therefore, only results for caps for which $\Phi = 90^\circ$ will be presented. For uniaxial extension, the results are valid irrespective of the angle $\Phi$, since the flow field is symmetric with respect to the $y$-axis. Shown in the graphs are the tensile component $\Sigma_T$ and the shear component $\Sigma_\Phi$ of the hydrodynamic force per unit of the base area ($\Sigma_\Phi = 0$ in all the cases under consideration).

Both tensile and shear components are made dimensionless on $\mu$.$\bar{C}$. In the case of the tensile component, a positive sign indicates tension (tensile force pointing away from the sphere) and a negative sign compression.

### 3.1. Angular orientation of the cap

The components of the dimensionless hydrodynamic force per unit area of the base of the cap are first shown as a function of the position of the spherical cap, i.e. as a function of the angle $\Theta$ since $\Phi$ is set to be $90^\circ$. For the sake of illustration, a dimensionless radius value of $x = 15$ was selected. This value was chosen in order to model real agglomerates characterized by $a \approx 1.5$ mm and $k \approx 0.01$ mm$^2$ used in past experiments (Levresse et
al., 1999). The results corresponding to simple shear flow can be found in Fig. 4: for caps for which $\Theta = 0, 90$ or $180^\circ$ the magnitude of the shear force is maximum while the tensile force is zero. For caps for which $\Theta = 45$ or $135^\circ$, the situation is reversed. The results for planar elongation and uniaxial extension are shown in Figs. 5 and 6, respectively. For caps for which $\Theta = 0$ or $180^\circ$ the tensile component of the hydrodynamic force per unit area of the base is similar for both planar elongation and uniaxial extension and is twice that obtained for simple shear flow for caps of similar size for which $\Theta = 45^\circ$. In the case of planar elongation, the
Influence of $\Theta$ on the hydrodynamic force per unit area of the base of the cap exerted by the fluid upon the solid in uniaxial extension flow ($x = 15$, $\delta = 5$, $\eta = 15$, any $\Phi$). The tensile component for caps for which $\Theta = 90^\circ$ is equal in magnitude but opposite in sign from that exerted on caps for which $\Theta = 0$ or $180^\circ$. In the case of uniaxial extension, caps for which $\Theta = 90^\circ$ exhibit a tensile component opposite in sign and half in magnitude from that obtained for caps for which $\Theta = 0$ or $180^\circ$. For both planar elongation and uniaxial extension, the magnitude of the shear force is maximum for $\Theta = 45$ or $135^\circ$, but at this position the tensile force is zero only in the case of the planar elongation geometry. The magnitude of the maximum shear force in the case of planar elongation is twice that obtained for simple shear flow for caps of similar size for which $\Theta = 0$, $90$ or $180^\circ$.

3.2. Size of the cap and thickness of the infiltrated layer

In Fig. 7, the dimensionless tensile component of the hydrodynamic force per unit area of the base of the cap is plotted as a function of the dimensionless height of the cap $\eta$ for porous spheres characterized by various infiltrated layer thicknesses. The dimensionless radius of the spheres again is chosen to be $x = 15$. The results presented in Fig. 7 correspond to planar elongation and uniaxial extension and are obtained for spherical caps for which the tensile force is maximized ($\Theta = 0$ or $180^\circ$). As indicated above, at this position the value of the tensile force is identical for the two flow fields. The results for simple shear can be simply deduced from those shown in Fig. 7. As discussed previously the maximum tensile force in the simple shear flow geometry (obtained for $\Theta = 45^\circ$) is half that for planar elongation and uniaxial extension (obtained for $\Theta = 0$ or $180^\circ$). The overall behavior is, therefore, similar for the three flow fields. The hydrodynamic tensile force per unit area of the base of the cap increases with both the thickness of the infiltrated layer and the height of the spherical cap. It should also be noted that the increment in the force becomes smaller as the infiltrated layer thickness increases.

In the case of the shear component of the hydrodynamic force the conclusions are different. Fig. 8 shows the maximum shear component in the case of the planar elongation flow ($\Theta = 135^\circ$). Results follow similar patterns for simple shear ($0$ or $180^\circ$) or uniaxial extension. Each curve of the shear force per unit area of the base of
the cap exhibits a maximum for a different cap size. The cap size that maximizes the shear force increases with the thickness of the infiltrated layer. Contrary to the case of the tensile force, a larger infiltrated layer at constant cap size does not necessarily enhance the shear force. These results for the shear component reflect a stronger influence of the slip boundary condition at the infiltrated shell/dry core interface by comparison with the tensile component.

3.3. Permeability

In order to be able to assess numerically the hydrodynamic force transmitted to the solid for very low permeability, fully filled porous spheres ($R = 0$) were analyzed. For the three flow fields, the hydrodynamic force for permeable spheres is compared to the hydrodynamic force at the surface of an impermeable sphere of identical radius. To mimic real agglomerates prepared in our lab, a sphere radius $a$ of 1.5 mm was chosen. The two spherical caps under consideration are hemispheres: cap 1 is characterized by $\Theta = 0$ and cap 2 by $\Theta = 45^\circ$.

The results corresponding to the case of simple shear, planar elongation and uniaxial extension are shown in Figs. 9, 10 and 11, respectively, for both cap 1 and cap 2, and for both the tensile and shear components of the hydrodynamic force per unit area of the base of the cap. Combinations missing from the graphs correspond to cases in which the component is zero for the cap under consideration. From the graphs it can be concluded that the magnitude of the hydrodynamic force per unit area of the base of the cap increases with the inverse of the permeability of the porous sphere and tends toward an asymptote. In the case of the shear force, the asymptotic value is identical to the force per unit area of the base of the cap exerted on the surface of an impermeable sphere. However, in the case of the tensile force there is a difference between the low permeability limit of a permeable sphere and the value corresponding to an impermeable sphere. This difference comes from the pressure exerted by the fluid inside the permeable sphere on the base plane of the cap (i.e. $\int_S (p - p_0) \, dS$). In the case of an impermeable sphere which does not contain any fluid, this term vanishes. The effect of this internal pressure is to provide an additional tensile force that tends to push the hemispheres apart.
4. Conclusions

The velocity and stress fields were calculated around porous spheres and within infiltrated spherical shells beneath their surface. This was accomplished for three particular far-field flows, but the general method of solution employed could be applied to any other flow field. The cumulative hydrodynamic force exerted by the fluid upon the solid was calculated for spherical caps with planar bases from the stress field within the porous sphere infiltrated shell. A comparison between the hydrodynamic forces acting on a spherical cap and the cohesive forces (van der Waals, liquid bridges, etc.) holding the cap to the agglomerate could provide information about the probability of detachment of the cap from the agglomerate, thus contributing to a better understanding of dispersion processes.

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Appendix A

Parameters corresponding to a partially filled porous sphere (R ≠ 0):

\[
E = \frac{5\beta^3/3}{\left(W - W' \left(2V/\beta^3 + 3W/\beta^5\right)\right)},
\]

\[
F = - \frac{(2V/\beta^3 + 3W/\beta^5)}{(2V'\beta^3 + 3W'/\beta^5)} E,
\]

where

\[
V = \left\{ (-30 + 10\beta^2 + \beta^4)\sinh \zeta + 30\beta \cosh \zeta \right\},
\]

\[
V' = \left\{ (-30 + 10\beta^2 + \beta^4)\cosh \zeta + 30\beta \sinh \zeta \right\},
\]

\[
W = (5\beta^2 + \beta^4)\sinh \beta - 5\beta^3 \cosh \beta + \frac{2\beta^5}{3\zeta} \sinh \zeta,
\]

\[
W' = (5\beta^2 + \beta^4)\cosh \beta - 5\beta^3 \sinh \beta + \frac{2\beta^5}{3\zeta} \cosh \zeta.
\]

\[
I = \frac{1}{2} \left\{ \frac{\beta^5}{2} - 3C^2 + 2D + E(6 + 3\beta^2)\sinh \zeta - (6\beta + \beta^3)\cosh \zeta \right\},
\]

\[
J = \frac{1}{12} (2D - 3C^2).
\]

Appendix B

Parameters corresponding to a fully filled porous sphere (R = 0):

\[
E = \frac{(5/12)\beta^3}{\left(\left[(5/\beta^3 + 5/3 + \beta^4/6)\sinh \zeta - (5/3) \cosh \zeta \right] \right)},
\]

\[
C = \frac{1}{6} - \frac{E}{15\beta^3} \sinh \zeta,
\]

\[
I = \frac{1}{2} \left\{ \frac{\beta^5}{2} - 3C^2 + E(6 + 3\beta^2)\sinh \zeta - (6\beta + \beta^3)\cosh \zeta \right\},
\]

\[
J = - \frac{\beta^5}{4C}.
\]

Appendix C

Parameters corresponding to an impermeable sphere:

\[
I = \frac{\beta^5}{4},
\]

\[
J = - \frac{5}{12} \beta^3.
\]

References


