Applicability and limitations of the phase-oscillator approximation in neuroscience

Roberto Fernández Galán
galan@cnbc.cmu.edu

with
Bard Ermentrout and Nathan Urban
Phase resetting and biological rhythms

Phase oscillator approximation: the Kuramoto model

\[ \frac{d\phi}{dt} = \omega \]
Phase oscillator approximation: the Kuramoto model

\[ \frac{d\phi}{dt} = \omega \]
Phase oscillator approximation: the Kuramoto model

\[
\frac{d\varphi}{dt} = \omega + Z(\varphi) \cdot I(t)
\]
Network dynamics of coupled phase oscillators

$$\frac{d\varphi_i}{dt} = \omega_i + \sum_{j \neq i}^{N} J_{ij} \cdot Z_i (\varphi_i - \varphi_j)$$

- intrinsic frequency
- relative phase
- phase-response curve
- synaptic sign and strength
Network of excitatory and inhibitory neurons

pattern 1

pattern 2

pattern 3
Some recent advances in applications of the iPRC to neuroscience


Applications of the iPRC

iPRC, $Z(\varphi)$
Applications of the iPRC

stability of the oscillations
Stability of the oscillations quantified through the Liapunov exponent

\[ \text{LE} = -\frac{1}{T} \int_{0}^{T} |Z'(t)|^2 dt \]

Applications of the iPRC

\[ \text{LE} = -\frac{1}{T} \int_{0}^{T} |Z'(t)|^2 dt \]
Applications of the iPRC

\[ LE = -\frac{1}{T} \int_0^T |Z'(t)|^2 dt \]
Relationship between the spike-triggered averaged and the neuron’s phase response curve

\[ \text{PRC}(\phi) = -\int_0^\phi \text{STA}(t) dt \quad \text{STA}(t) = -\frac{d}{dt} \text{PRC}(t) \]

Ermentrout et al., submitted
Applications of the iPRC

\[ LE = -\frac{1}{T} \int_0^T |Z'(t)|^2 dt \]

\[ STA = -\frac{dZ(t)}{dt} \]
Applications of the iPRC

iPRC, $Z(\varphi)$

LE = $-\frac{1}{T} \int_0^T |Z'(t)|^2 dt$

STA = $-\frac{dZ(t)}{dt}$

synchronized assemblies
Clustering of neural oscillators


diagram showing excitatory and inhibitory synapses in initial and steady states.

\[ \lambda_n = J \pi n (S_n - i \omega C_n) \]

Applications of the iPRC

\[ \lambda_n = J \pi n \left( S_n - i \omega C_n \right) \]

\[ \text{LE} = -\frac{1}{T} \int_0^T |Z'(t)|^2 dt \]

\[ \text{STA} = -\frac{dZ(t)}{dt} \]
Applications of the iPRC

iPRC, $Z(\varphi)$

$\lambda_n = J \pi n (S_n - i \omega C_n)$

$\text{LE} = -\frac{1}{T} \int_0^T |Z'(t)|^2 dt$

$\text{STA} = -\frac{dZ(t)}{dt}$

optimal time scale of spike-time reliability
Optimal time-scale for spike-time reliability

\[
\mu \approx -\frac{1}{T} \int_0^T Z''(t) \int_0^t Z(s) C(t-s) ds dt
\]

\[
C(t) = C_0 \exp\left(-\frac{|t|}{\tau}\right)
\]

Liapunov exponent, \( \mu \)
for the phase difference:

Galán et al., submitted
Applications of the iPRC

\[ \mu \approx -\frac{1}{T} \int_0^T Z''(t) \int_0^t Z(s) C(t-s) ds \, dt \]

\[ \lambda_n = J \pi n (S_n - i \omega C_n) \]

\[ \text{LE} = -\frac{1}{T} \int_0^T |Z'(t)|^2 \, dt \]

\[ \text{STA} = -\frac{dZ(t)}{dt} \]
Applications of the iPRC

\[ \lambda_n = J \pi n (S_n - i\omega C_n) \]

\[ \mu \approx -\frac{1}{T} \int_0^T Z''(t) \int_0^t Z(s)C(t-s)dsdt \]

\[ \text{LE} = -\frac{1}{T} \int_0^T |Z'(t)|^2 dt \]

\[ \text{STA} = -\frac{dZ(t)}{dt} \]

stochastic synchronization
integrators

resonators

R.F. Galán et al. (2007) *Neurocomputing*, 70, 2102
Applications of the iPRC

- stochastic synchronization
- optimal time scale of spike-time reliability
- iPRC, $Z(\varphi)$
- stability of the oscillations
- optimal stimulus
- synchronized assemblies
Applicability and Limitations

The iPRC tells us about the:

- Stability of the oscillations (LE)
- Optimal stimulus (STA)
- Ability to synchronize in a network
- Preferred time scales of the inputs
- Influence of membrane properties on stochastic synchronization

The iPRC approach is limited when:

- The oscillators are strongly perturbed
- Synchronization is not purely oscillatory like in “winnerless competition”