Predicting synchronized neural assemblies from experimentally estimated phase-response curves

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Assemblies of synchronized neurons have been observed in a variety of neural systems (e.g. Harris, 2003; Plenz & Aertsen, 1996; Wehr & Laurent, 1996).

These neural assemblies are thought to encode sensory information or store short-term memories.

This phenomenon is reminiscent of the formation of clusters in models of coupled phase oscillators (e.g. Kuramoto, 1986; Golomb et al., 1992; Crawford, 1995).
Phase model of a spiking neuron

\[
\frac{d \varphi}{dt} = \frac{2\pi}{T} = \omega
\]
Phase model of coupled neurons

\[ \frac{d\varphi_1}{dt} = \omega_1 + J_{12} \cdot H_1(\varphi_1 - \varphi_2) \]

\[ \frac{d\varphi_2}{dt} = \omega_2 + J_{21} \cdot H_2(\varphi_2 - \varphi_1) \]
Network dynamics of coupled phase oscillators

\[ \frac{d\phi_i}{dt} = \omega_i + \sum_{j \neq i}^N J_{ij} \cdot H_i(\phi_i - \phi_j) \]

- intrinsic frequency
- relative phase
- synaptic sign and strength
- interaction function approximated by the phase-response curve
Phase-response curves
Phase-response curves
Phase response in mitral cells

Resonance in mitral cells

R. F. Galán et al., in preparation
Network dynamics of coupled phase oscillators

- **Excitatory** connections
- **Inhibitory** connections
Clustering of neural oscillators

- **Excitatory synapses**
- **Inhibitory synapses**
Network dynamics of coupled phase oscillators

Continuum approximation in homogenous networks of phase oscillators ($N \to \infty$, $\omega_{ij} = \omega$, $J_{ij} = J$, $H_i = H$)

$$\frac{d\varphi}{dt} = \omega + J \int_{0}^{2\pi} H(\varphi - \theta) \rho(\theta) d\theta$$

Continuity equation for the oscillator density, $\rho$

$$\frac{d\rho}{dt} = -\frac{\partial}{\partial \varphi} \left[ \rho \left( \omega + J \int_{0}^{2\pi} H(\varphi - \theta) \rho(\theta) d\theta \right) \right]$$
Symmetry breaking and emergence of synchronized assemblies

The linearization of the continuity equation around an initial uniform state reads:

\[
\frac{dy}{dt} = -\omega \frac{\partial y}{\partial \phi} - J \frac{\partial y}{\partial \phi} \int_{0}^{2\pi} H(\phi - \theta) y(\theta) d\theta
\]

whose solutions are associated with the eigenvalues:

\[
\lambda_n = J \pi n (S_n - i \omega C_n)
\]

Sine coefficients \quad Cosine coefficients
Symmetry breaking and emergence of synchronized assemblies

\[ PRC(\theta) \approx H(\theta) = \sum_n \left( C_n \cos n\theta + S_n \sin n\theta \right) \]

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Unstable mode for \textit{excitatory} synapses
Symmetry breaking and emergence of synchronized assemblies

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Unstable modes for **inhibitory** synapses
Clustering of neural oscillators

excitatory synapses

inhibitory synapses

initial state

steady state
Summary
Experimental estimation of phase-response curves in mitral cells reveals type II excitability (resonators)
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Summary

- Experimental estimation of phase-response curves in mitral cells reveals type II excitability (resonators).

- Higher order harmonics of the phase-response curve are necessary for the formation of more than one synchronized neural assemblies.

- Positive sine coefficients break the symmetry of the homogeneous state leading to cluster formation.

- Inhibitory interactions between mitral cells lead to the formation of a dominant synchronized neural assembly in the gamma frequency band.
Final words
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Alternatively, synchronized assemblies may emerge from a symmetry breaking transition determined by the neural phase response.
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Final words

- The formation of synchronized neural assemblies does **not** require short-term **plasticity**

- Alternatively, synchronized assemblies may emerge from a **symmetry breaking** transition determined by the neural phase response

- Equidistant oscillator clusters may account for different **frequency bands** (gamma band \( \approx 2 \cdot \) beta band)

- Do cell assemblies represent a **neural code**, or are they a by-product of the network connectivity?
Thanks for your attention

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