Math 224 Exam 1

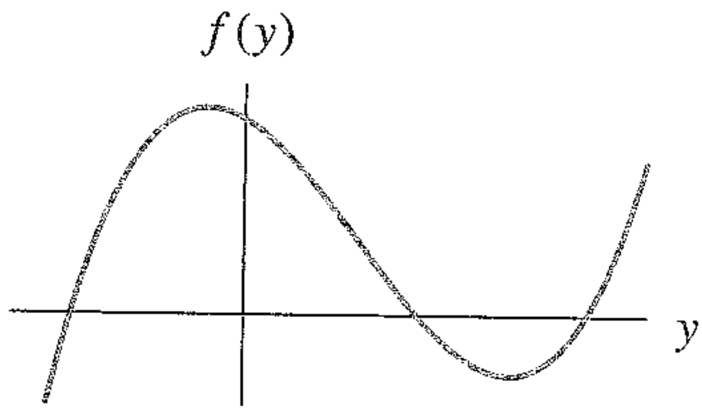
September 12, 2012

1. (a) Solve the initial value problem

$$\frac{dy}{dt} = y^2 t, \qquad y(0) = 1.$$

   (b) What is the domain of definition of your solution? What happens as t approaches the limits of that domain?

2. Suppose the following is a graph of the function $f(y)$.



Sketch the slope field of the differential equation

$$\frac{dy}{dt} = f(y),$$

and describe the possible long term behaviors of the solutions, depending on their initial conditions.

(**Suggstion**: label important points on the axes of the graph above.)

3. (a) State the Euler's method formula for y_{k+1} in terms of t_k, y_k and Δt when approximating the solution to the initial value problem

$$\frac{dy}{dt} = f(t, y) \quad y(t_0) = y_0.$$

Sketch a graph that demonstrates where this formula comes from.

(b) Use Euler's method with $\Delta t = 1$ to approximate the solution of

$$\frac{dy}{dt} = 2y - t, \qquad y(0) = 0$$

up to $t = 3$.

4. The behavior of the population of deer in a particular wooded area is modeled by the logistic equation

$$\frac{dP}{dt} = \frac{1}{10}P\left(1 - \frac{P}{2}\right),$$

where P is the population in thousands, time is measured in days.

(a) Suppose that a fraction α of the population is hunted each day; modify the differential equation to reflect this.

(b) Suppose first that $0 < \alpha < \frac{1}{10}$.

i. Find all equilibria and sketch the phase line. Include only the relevant range $P > 0$.

ii. Use qualitative analysis to predict the fate of the deer population if the population is initially 5 thousand deer (i.e., $P(0) = 5$).
(**Note**: Even though the equilibrium from the previous part is in terms of α, you can still tell how it compares to 5.)

(c) Suppose now that $\alpha > \frac{1}{10}$.

i. Find all equilibria and sketch the phase line. Include only the relevant range $P > 0$.

ii. Use qualitative analysis to predict the fate of the deer population if the population is initially 5 thousand deer (i.e., $P(0) = 5$).

(d) What's so special about the value $\alpha = \frac{1}{10}$? That is, how does the nature of the system change as α passes through that value?