Name: ______________________________

Math 224 Exam 2

September 28, 2012

1. Consider the predator-prey system

$$\frac{dR}{dt} = 2R - 1.2RF,$$
$$\frac{dF}{dt} = -F + 0.9RF.$$

   (a) Suppose that the predators find a second food source in limited supply. How would you modify the system to take this into a account?

   (b) Suppose that predators migrate into the area at a constant rate if there are at least ten times as many prey as predators in the area (that is, if $R > 10F$), and they move away at a (possibly different) constant rate if there are fewer than ten times as many predators. How would you modify the system to take this into account? Possibly useful notation:

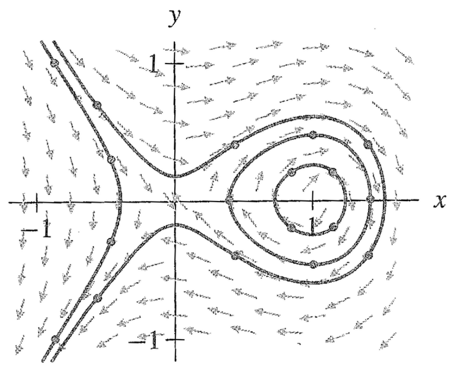
$$\mathbb{1}(x > 0) = \begin{cases} 1 & x > 0; \\ 0 & x \leq 0; \end{cases} \qquad \textit{and} \qquad \mathbb{1}(x < 0) = \begin{cases} 1 & x < 0; \\ 0 & x \geq 0. \end{cases}$$

2. Solve the system

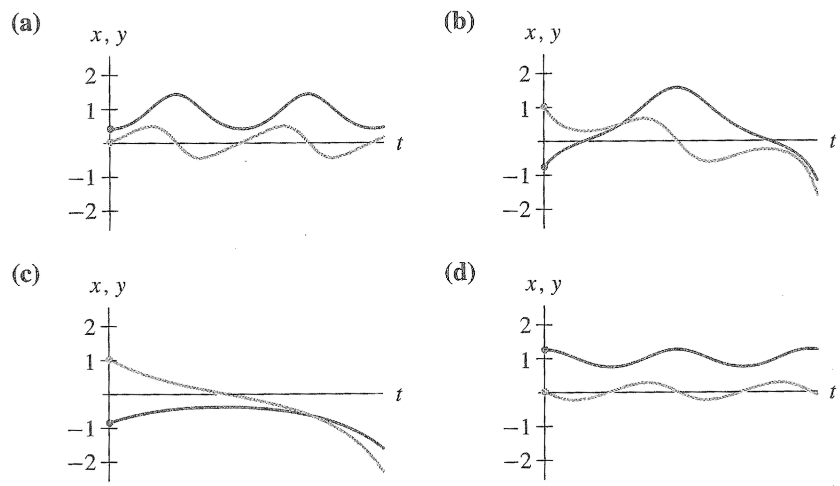
$$\frac{dx}{dt} = 3x + y,$$
$$\frac{dy}{dt} = -y$$

with initial conditions $x(0) = 1$, $y(0) = 2$.

3. The following is a graph of four solution curves $(x(t), y(t))$ to an autonomous system of differential equations, together with the direction field of the system.



Below are four pairs of graphs of $x(t)$ and $y(t)$ versus t.



Match each of the pairs of graphs to a solution curve in the phase plane. Label which graph is x and which is y. Finally, describe the long-term behavior of solutions in all cases.

4. Find two nonzero solutions of the differential equation

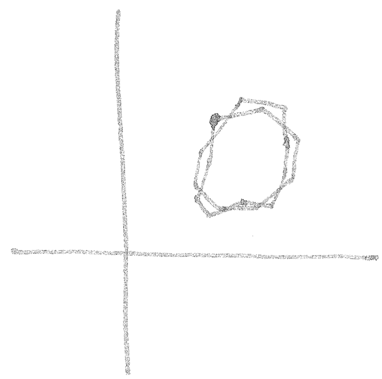
$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 6y = 0$$

which are not constant multiples of each other.

5. Suppose you used Euler's method to approximate the solution to the **autonomous** system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{F}(\mathbf{Y})$$

with initial condition $\mathbf{Y}(0) = \mathbf{Y}_0$, and the resulting solution curve plotted on the phase plane looked like this:



(a) Explain how you can tell that the Euler's method approximation must not be a very good approximation of the true solution.

(b) What would you do to try to get a better approximation?

(c) What do you guess the true solution looks like, based on the approximation above?