

Use Theorem 2.23 from the book to prove the following.

Theorem 1. Let B_1, \dots, B_k be linearly independent $n \times n$ matrices over \mathbb{R} such that $\text{tr}(B_i B_i^t) = n$ for each i . Let $b_{ij} = \text{tr}(B_i B_j^t)$. Let M be a random orthogonal matrix and let

$$W = (\text{tr}(B_1 M), \text{tr}(B_2 M), \dots, \text{tr}(B_k M)) \in \mathbb{R}^k.$$

Let $Y = (Y_1, \dots, Y_k)$ be a random vector whose components are standard normals, with covariance matrix $\frac{1}{n} (b_{ij})_{i,j=1}^k$. Then

$$\sup_{\|f\|_L \leq 1} |\mathbb{E}f(W) - \mathbb{E}f(Y)| \leq \frac{2\sqrt{\lambda}k}{n-1}$$

for $n \geq 3$, where λ is the largest eigenvalue of $\frac{1}{n} (b_{ij})_{i,j=1}^k$ and $\|f\|_L$ is the Lipschitz norm of f .