Proof part 2 of Proposition 4.7 for  $\mathbb{SO}(2N).$