Let $\{\mathcal{F}_n\}_{\geq 1}$ be a filtration and τ a stopping time. Recall that

$$\mathcal{F}_\tau = \{A \in \mathcal{F} : A \cap \{\tau \leq k\} \in \mathcal{F}_k, 1 \leq k < \infty\}.$$

Show that $\mathbb{1}_A(\omega) = \mathbb{1}_A(\omega')$ for all $A \in \mathcal{F}_\tau$ if and only if $\tau(\omega) = \tau(\omega')$ and $X_j(\omega) = X_j(\omega')$ for $j = 1, 2, \ldots, \tau(\omega)$.