1. Let $B(t)$ be a standard Brownian motion, let $x \in \mathbb{R}$ and let $T := \inf\{t > 0 : B(t) = x\}$. For $0 < s < t$, show that

$$\mathbb{P}[T \leq s, \sup_{0 \leq r \leq t} B(r) \leq x] = 0.$$

2. Let $B(t)$ be a standard Brownian motion. Show that $B(t)^2 - t$ is a martingale.

3. Let $B(t)$ be a standard Brownian motion. Show that the following transformations of B are also Brownian motions (that is, they have independent centered Gaussian incremets with variance given by the length of the interval, and they have continuous sample paths):

   (a) $B_c(t) := \frac{1}{c} B(c^2 t)$, where $c > 0$

   (b) $\tilde{B}(t) := \begin{cases} tB\left(\frac{1}{t}\right), & t > 0; \\ 0, & t = 0. \end{cases}$