Use Lévy's lemma (see section 5.1) together with Borel's lemma (section 2.1) to prove Gaussian measure concentration for Lipschitz functions:

Let $Z = (Z_1, \ldots, Z_k)$ be a standard Gaussian random vector. Let $f : \mathbb{R}^k \to \mathbb{R}$ be Lipschitz with Lipschitz constant L, and let M be such that $\mathbb{P}[f(Z) \geq M] \geq \frac{1}{2}$ and $\mathbb{P}[f(Z) \leq M] \geq \frac{1}{2}$. Then there are constants c, C such that

$$\mathbb{P}[|f(Z) - M| \geq Lt] \leq Ce^{-ct^2}.$$