Modeling transmission of infection in a distributed heterogeneous environment: Schistosomiasis control

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Schistosomiasis

- Chronic parasitic trematode infection
- 200-300 million people worldwide
- Significant morbidity (esp. anemia)
- Premature mortality
- Life-cycle is complex, requiring species-specific intermediate snail host
- Optimal control strategies have not been established.

Epidemiology

- Transmission highly focal
- Prevalence and intensity vary with age
- Morbidity varies with age
- Infection intensity not randomly dispersed
- Signs of early chronic disease correlated with greater intensity of infection

Results of some field studies

Impact of age-targeted chemotherapy on prevalence and intensity of contact rate of infection as a function of time (Fig. 3).

Mathematical model

Simplest model: 1 human and 1 snail population (2 variables)

\[ \begin{align*}
  w_{ia} &= \frac{w_{ia} - \rho_{ia} \cdot t}{H_{ia} + r \cdot t} \\
  w_{ia} &= \frac{w_{ia} - \rho_{ia} \cdot t}{H_{ia} + r \cdot t} \\
  A &= \text{transmission coefficient (Snail-to-Human, Human-to-Snail)}
\end{align*} \]

Endemic equilibria \( \left( \rho, \rho \right) \) exist, iff the Basic Reproduction Number

\[ R_0 = \frac{A \cdot \sigma}{\beta} \]

Figure 1: Model environment made of 12 human clusters and 10 snail sites with selected marked distances (top), along with population functions and snail densities (bottom).

General system accounts for (i) age structure and aging, (ii) age dependent contact rate, (iii) age targeted chemotherapy, (iv) age dependent and (v) age specific mortality.

Parameters \( A \): population fraction, \( \rho \): contact rate of infection, \( \sigma \): probability of worm establishment per contact, \( \beta \): age dependent transmission rate, \( \rho \): age bin time (step). The effect of chemotherapy is increased worm mortality: \( \rho \rightarrow \rho + r \cdot \rho \) (age dependent).

Epidemiology

- Environmental modification;
- Vaccination;
- Environmental modification;
- Development

Was Transmission Affected?

Partial success, depending on location

Figure 2: Community worm burden \( n_{ia} \) as function of age class, before treatment (black), and with uniform (across age) treatment at frequency \( f = 0.5, 1 \text{yr} \) (shows substantial reduction).

Conclusions

- Schistosomiasis infection in highly endemic areas can be mathematically modeled via a distributed "broad-spectrum" (Maiden/McDonald) system.
- We place it in a heterogeneous environment made of human and snail clusters with age-stratified populations and behaviors.
- We study endemic equilibrium in such systems and their dependence on the essential control parameters: snail densities and worm attrition rates. The latter, in particular allows an optimal control strategy of the community "mean worm burden" via repeated (cyclic) chemotherapy.
- For age targeted chemotherapy (high risk groups) the computed optimal solutions show significant reduction of the mean worm burden at a relatively low cost.
- On the contrary site specific control (of high risk sites) proves largely inefficient, which can be explained by an overall connectivity pattern of the human-snail.