$$\alpha - \alpha R_0^2 = 2R_0^2(1-\alpha)$$

$$\alpha = R_0^2\left[2 - 2\alpha + \alpha\right]$$

$$= R_0^2\left[2-\alpha\right]$$

$$R_0^2 = \frac{\alpha}{2-\alpha}$$

$$R_0 = \sqrt{\frac{\alpha}{2-\alpha}}.$$

Fishy factor of 2.

Looks like Petz-Raffy

$Q(z)$ is our $\frac{V(z)}{2}$.

Want R_0 smallest solution

to $R_0 Q'(R_0) = 1$

$$\frac{R_0 V'(R_0)}{2}$$

$$\Leftrightarrow \quad R_0 V'(R_0) = 2$$

Now, $V'(r) = \frac{2r(1-\alpha)}{\alpha(1-r^2)}$

$\Rightarrow$ Want

$$\frac{\cancel{2}R_0^2}{(1-R_0^2)}\left(\frac{1-\alpha}{\alpha}\right) = \cancel{2}$$

$$\Leftrightarrow \quad R_0^2\left(\frac{1-\alpha}{\alpha}\right) = 1-R_0^2$$

$$R_0^2\underbrace{\left[\frac{1-\alpha}{\alpha}+1\right]}_{=\frac{1}{\alpha}} = 1$$

$$\Rightarrow \quad R_0^2 = \alpha$$

$$R_0 = \sqrt{\alpha}$$

good.

(Scratch Work sample 1/6.)

$$= \frac{2(1-\alpha)^2}{\alpha^3}\left[\frac{1}{2}\int_0^{\infty} -\frac{w\log w}{(1-w)^3}\,dw - \frac{1}{2}\log\alpha\left(-\frac{\alpha^2}{2(1-\alpha)^2}\right)\right]$$

$$g(x) = \frac{x^2\log x + x - (x-1)^2\log(1-x) - 1}{2(x-1)^2}$$

$$g'(x) = \frac{2(x-1)^2\left[2x\log x + \cancel{x} + \cancel{1} - 2(x-1)\log(1-x)\ldots\right] - \left[x^2\log x + x - (x-1)^2\log(1-x) - 1\right]4\ldots}{4(x-1)^4}$$

$$= \log x\left[4x(1-x)^2 - 4x^2(x-1)\right]$$

$$+ \log(1-x)\left[-4(x-1)^3 + 4(x-1)^3\right]$$

$$+ \cancel{2(x-1)^2(x+1) - 2(x}$$

$$-4(x-1)^2$$

$$= -\frac{(1-\alpha)^2}{\alpha^3}\left[\frac{w^2\log w + w - (1-w)^2\log(1-w) - 1}{2(1-w)^2}\Bigg|_0^{\alpha} - \log\alpha\left(\frac{\alpha^2}{2(1-\alpha)^2}\right)\right]$$

$$= -\frac{(1-\alpha)^2}{\alpha^3}\left[\frac{\alpha^2\log\alpha + \alpha - (1-\alpha)^2\log(1-\alpha) - 1}{2(1-\alpha)^2} + \frac{1}{2} - \frac{\alpha^2\log\alpha}{2(1-\alpha)^2}\right]$$

$$= -\frac{(1-\alpha)^2}{\alpha^3}\left[\frac{-(1-\alpha) - (1-\alpha)^2\log(1-\alpha) + (1-\alpha)^2}{2(1-\alpha)^2}\right]$$

$\frac{1}{R^2}\log\left(\frac{1}{R}\right) = -\frac{\log R}{R^2} \to 0$

$$= \frac{(1-\alpha)}{2\alpha^3}\left[1 + (1-\alpha)^2\log(1-\alpha) - (1-\alpha)\right]$$

(Scratch Work sample 2/6.)

$$\sum_{j=1}^{m} \frac{(n-m)_j}{(j-1)!}(1-u)^{j-1} = \sum_{j=0}^{m-1} \frac{(n-m)_{j+1}}{j!}(1-u)^j = (n-m)\sum_{j=0}^{m-1} \frac{(n-m+1)_j}{(1)_j}(1-u)^j$$

$$= (n-m)\sum_{j=0}^{m-1} \binom{m-1}{j} \frac{(n-m+1)_j (m-1-j}{\cancel{(m-1)!}\ (m-1)!}$$

$$= (n-m)\sum_{j=0}^{m-1} \binom{m-1}{j} \frac{(n-m+1)_j}{(m-1)\cdots(m-j)}(1-u)^j$$

$${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}$$

$${}_2F_1(-m,b;c;z) = \sum_{n=0}^{m} (-1)^n \binom{m}{n} \frac{(b)_n}{(c)_n} z^n$$

$$\sum_{j=0}^{m-1} [n-(m-1)]_j \left[\frac{(1-u)^j}{j!}\right]$$

If $b = c$...

$$\sum_{\ell=m}^{n-(m-1)} n-m+$$

$$\sum_{\ell=0}^{m-1} \frac{(n-(m-1))_\ell}{\ell!}(z\bar{w})^\ell$$

$$\sum_{\ell=0}^{n-(m-1)} \binom{n-(m-1)}{\ell}\cancel{(z\bar{w})^\ell = (1+z\bar{w})^{n-(m-1)}}$$

so what? This isn't helpful.

$${}_pF_q(a_1,\ldots,a_p;\ b_1,\ldots,b_q;\ z) = \sum_{n=0}^{\infty} \frac{(a_1)_n\cdots(a_p)_n}{(b_1)_n\cdots(b_q)_n}\frac{z^n}{n!}$$

$$\sum_{\ell=0}^{m-1} \frac{(n-(m-1)+\ell-1)!}{\ell!\,(n-(m-1)-1)!}(z\bar{w})^\ell = \sum_{\ell=0}^{m-1} \binom{n-m+\ell}{\ell}$$

(Scratch Work sample 3/6.)

$i \neq q,\ j \neq \ell$:

$$\frac{1}{\binom{n}{k}} \sum_S \sum_{\substack{j,\ell \in S \\ j \neq \ell}} a_{i\ell} a_{qj} = \frac{1}{\binom{n}{k}} \sum_{\substack{r \neq s \\ 1 \le r,s \le n}} a_{ir} a_{qs} \binom{n-2}{k-2}$$

$$= \frac{k(k-1)}{n(n-1)} \Big[\Big(\sum_r a_{ir}\Big)\Big(\sum_s a_{qs}\Big) - \underbrace{\sum_r a_{ir} a_{qr}}_{= \delta_{iq}} \Big]$$

$j = \ell$:

$$\frac{1}{\binom{n}{k}} \sum_S \sum_{j \in S} a_{ij} a_{qj} = \frac{1}{\binom{n}{k}} \sum_{j=1}^{n} a_{ij} a_{qj} \cdot \binom{n-1}{k-1}$$

$$= \frac{k}{n} \delta_{iq}$$

$$\mathbb{E}\left[A T_{S'} u_2 - A T_S u \mid S, u\right]$$

$$\mathbb{E}\left[A T_S (u_2 - u) \mid S, u\right] = -\frac{\varepsilon^2}{2k} A T_S u$$

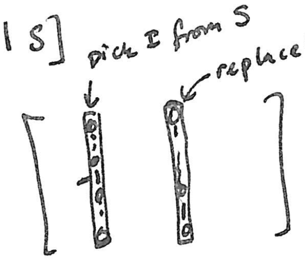
$$= A \Big[\mathbb{E}\left[T_{S'}(u_2 - u) \mid S, u\right] + \mathbb{E}\left[(T_{S'} - T_S) u \mid S, u\right] \Big]$$

$$= A \Big[-\frac{\varepsilon^2}{2k} \mathbb{E}\left[T_{S'} \mid S\right] \cdot u + \left(-\frac{1}{k} T_S u\right) + \left(\frac{1}{n-k}\right)(I - T_S) u \Big]$$

What about pair of S's

$$\mathbb{E}\left[T_{S'} - T_S \mid S\right]$$

$T_{S'} - T_S$:



$$= A \Big[-\frac{\varepsilon^2}{2k} \Big[\Big(\frac{k-1}{k}\Big) T_S + \frac{1}{n-k}(I - T_S) \Big] u - \frac{1}{k} T_S u + \Big(\frac{1}{n-k}\Big)(I - T$$

$$= A \Big[\Big(-\frac{\varepsilon^2 (k-1)}{2k^2} + \frac{\varepsilon^2}{2k(n-k)} - \frac{1}{k} - \frac{1}{n-k} \Big) T_S u$$

$$\mathbb{E}\left[T_{S'} - T_S \mid i, S\right] = \frac{1}{n-k} \left[I - T_S \right] - \left(I^{i\text{th}} \text{ col}\right)^T \Big(-\frac{\varepsilon^2}{2k(n-k)} + \frac{1}{n-k} \Big) u$$

$$\Rightarrow \mathbb{E}\left[T_{S'} - T_S \mid S\right] = \underbrace{\frac{1}{n-k}(I - T_S)}_{\text{error.}} - \frac{1}{k} T_S$$

not exactly.

(Scratch Work sample 4/6.)

$$\underline{\delta_n C_\varepsilon L_\varepsilon \approx \delta_n \left(\frac{2\pi-\theta}{2\pi}\right) n \log(\varepsilon e^{\lambda/n\delta_n})} \approx \frac{2\pi-\theta}{2\pi} \ ?$$

lower bound is

$$\delta_n \left[A_\varepsilon \cdot L_\varepsilon + B_\varepsilon \cdot L_{1-\varepsilon}\right]$$

$$\approx \frac{\theta}{2\pi} n \cdot \delta_n \cdot \frac{\lambda}{n\delta_n} = \frac{\theta\lambda}{2\pi}.$$

want actually $\delta_n C_\varepsilon L_\varepsilon$ to be small.

$\cancel{\delta_n \left(\frac{n\theta}{2\pi}\right)}$ $\quad \delta_n \cdot n \cdot L_\varepsilon$

$\approx \delta_n \cdot n \log\left(1 + \varepsilon\left(e^{\frac{\lambda}{n\delta_n}} - 1\right)\right)$

$\downarrow$

0 (maybe pretty fast)

$\cancel{C_\varepsilon = \{k : p_k \leq \varepsilon\}}$

$C_\varepsilon \leq n.$

so could still have ε s.t. $\varepsilon\left(e^{\frac{\lambda}{n\delta_n}} - 1\right)$ blows up

$$\log \varepsilon = \frac{-\lambda}{n\delta_n} + \frac{\xi(n)}{n\delta_n}$$

$$\varepsilon = e^{-\frac{\lambda}{n\delta_n} + \frac{\xi(n)}{n\delta_n}}$$

$\varepsilon(1-\varepsilon) = \left(e^{\frac{\xi(n)-\lambda}{n\delta_n}}\right)\left(1 - e^{\frac{\xi(n)-\lambda}{n\delta_n}}\right)$ $\cancel{\log \varepsilon \pm \lambda}$ $\left(\log \varepsilon + \frac{\lambda}{n\delta_n}\right) n\delta_n$

need $\log \varepsilon = \frac{-\lambda}{n\delta_n} + x$

s.t. $x n \delta_n \to 0$

$$\delta_n A_\varepsilon L_{1-\varepsilon} \leq \cancel{\delta_n} \frac{\lambda}{n\cancel{\delta_n}} \cdot \frac{\log n}{\varepsilon(1-\varepsilon)}$$

$$= \lambda \left(\frac{\log n}{n}\right)\left(\frac{1}{\varepsilon(1-\varepsilon)}\right)$$

$\frac{\varepsilon(n)}{n\delta_n}$

$$\approx \frac{\lambda \left(\frac{\log n}{n}\right)}{\varepsilon} = \lambda e^{\log\log n - \log n + \frac{(\xi(n)-\lambda)}{n\delta_n}}$$

$\xi(n) < \lambda + n\delta_n(\log$

need $\frac{\xi(n)-\lambda}{n\delta_n} < \log n$ $(+)$

(Scratch Work sample 5/6.)

$$\frac{\int_t^1 (r^2)^{j-1}(1-r^2)^{n-m-1}\, r\,dr}{\int_0^1 (r^2)^{j-1}(1-r^2)^{n-m-1}\, r\,dr}.$$

$$\int_0^t r^{2j-1} \cancel{(1-r^2)} \sum_{k=0}^{n-m-1} \binom{n-m-1}{k} \cancel{(} (-1)^k r^{2k} \frac{dr}{t} \cdot t$$

$$s = \frac{r}{t} \qquad ds = \frac{dr}{t}$$

$$= t \int_0^1 (ts)^{2j-1} \sum_{k=0}^{n-m-1} \binom{n-m-1}{k} (-1)^k (ts)^{2k}\, ds \quad \text{no use.}$$

$$m^{-\frac{1}{2}(n-m-1)} \log m$$

$$(ts)^{2j-1} (1-(ts)^2)^{n-m-1}$$

as long as
$n-m-1>0$, fine!

upper bound ?

$$\int_{B_{1-t}^c} (|z|^2)^{j-1} \underbrace{(1-|z|^2)^{n-m-1}}_{\le t^{n-m-1}} d\mu(z) \le t^{n-m-1} 2\pi \int_t^1 r^{2j-1}\, dr$$

$$= 2\pi\, t^{n-m-1} \left(\frac{1-t^{2j}}{2j} \right).$$

Sum over m: $\sum_{j=1}^{m} \frac{1}{2j} \sim \frac{1}{2} \log m \quad \sum_{j=1}^{m} \frac{t^{2j}}{2j} \le \frac{1}{1-t}$ not so great, actually

wanted $t = \frac{1}{Nm}$ $= \cancel{\ldots}$

(Scratch Work sample 6/6.)