Name: _____

Math 224 Exam 6
December 5, 2012

1. For $x, y > 0$, consider the system

$$\frac{dx}{dt} = \frac{1}{y}$$
$$\frac{dy}{dt} = \frac{a}{x},$$

where a is a real parameter.

(a) Show that the system is Hamiltonian and identify the Hamiltonian function.

(b) Sketch the level curves of the system for $a = \frac{1}{2}, 1, 2$, and use them to draw the phase portraits of the corresponding systems. (Note: it will be useful to recall that $\ln(b) - \ln(c) = \ln\left(\frac{b}{c}\right)$ and that $b\ln(c) = \ln(c^b)$.)

2. Consider the forced harmonic oscillator

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 17y = 0; \qquad y(0) = 1, \quad y'(0) = 0.$$

(a) Is the system overdamped or underdamped?

(b) Suppose you hit the oscillator with a hammer at time $t = 3$. Modify the equation above to reflect this and then solve it.

(c) How would your answer change if you hit the oscillator twice as hard?

3. Find the general solution to

$$\frac{dy}{dt} + 9y = u_2(t).$$

$$\mathcal{L}[y] = \int_0^\infty y(t)e^{-st}\,dt$$

$$\mathcal{L}[y'] = s\mathcal{L}[y] - y(0)$$

$$\mathcal{L}[y''] = s^2\mathcal{L}[y] - sy(0) - y'(0)$$

$y(t)$	$Y(s) = \mathcal{L}[y]$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$u_a(t)$	$\frac{e^{-sa}}{s}$
δ_a	e^{-as}
$u_a(t)f(t-a)$	$e^{-as}F(s)$
$e^{at}f(t)$	$F(s-a)$
$tf(t)$	$-\frac{dY}{ds}$