Name____

Math 224 Exam 1

January 30, 2015

1. (a) Find the general solution of $\dfrac{dy}{dt} = 2ty$.

(b) Verify that $y(t) = (\sin t)e^{t^2}$ is a solution of

$$\frac{dy}{dt} = 2ty + (\cos t)e^{t^2}.$$

(c) Find the solution of the initial value problem

$$\frac{dy}{dt} = 2ty + (\cos t)e^{t^2}, \qquad y(0) = 4.$$

2. Consider the modified logistic model

$$\frac{dP}{dt} = P\left(1 - \frac{P}{4}\right)\left(\frac{P}{2} - 1\right).$$

(a) Find all the equilibrium solutions of this differential equation.

(b) Draw the phase line for this equation, and identify the equilibria as sinks, sources, or nodes.

(c) What will be the long-term behavior of the population in each of the following situations?

i. $P(0) = 1$

ii. $P(0) = 3$

iii. $P(0) = 5$

(d) What is the significance, for a population described by this model, of the population size 2?

3. (a) Verify that $y_1(t) = \frac{1}{\sqrt{2t+1}}$ and $y_2(t) = \frac{1}{\sqrt{2t+4}}$ are both solutions of

$$\frac{dy}{dt} = -y^3.$$

(b) *Without* finding the general solution of the differential equation, what can you say about solutions of $\dfrac{dy}{dt} = -y^3$ for which the initial condition $y(0)$ satisfies $\frac{1}{2} < y(0) < 1$?

4. (a) Use Euler's method with $\Delta t = 1$ to approximate the solution of the initial value problem $\frac{dy}{dt} = (2 - y)(y + 1)$, $y(0) = 1$ at $t = 3$.

(b) Does your approximation seem reasonable? Why or why not?