

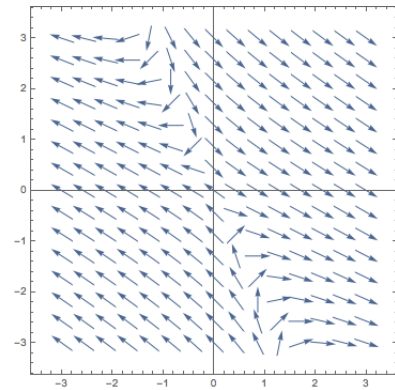
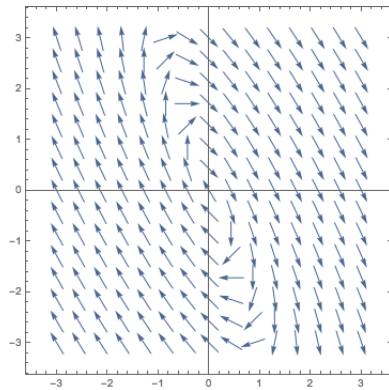
Name: _____

Math 224 Exam 3
March 6, 2015

1. Here are two different versions of the model for Paul and Bob's cafés, both of which suppose that current profits from either café have a positive effect on Paul's profits, and current profits from either café have a negative effect on Bob's profits:

$$\begin{aligned} \frac{dx}{dt} &= 5x + 2y & \frac{dx}{dt} &= 2x + y \\ \frac{dy}{dt} &= -3x - 2y & \frac{dy}{dt} &= -4x - y. \end{aligned}$$

- (a) Identify which of the following two direction fields is for which system. Justify your answer.



- (b) For each system, sketch the solution curve with initial condition $x(0) = 1, y(0) = 0$ on the direction field above. Describe the long-term behavior of the solutions.

2. Consider the system $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$, where $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$.

(a) Find the eigenvalues of \mathbf{A} .

(b) Determine from the eigenvalues alone what type of equilibrium the system has at the origin.

(c) Find the general solution of the system.

(d) Find the solution of the system with the initial condition $\mathbf{Y}(0) = (1, 0)$.

- (e) Sketch the phase portrait, including the solution curve with the initial condition $\mathbf{Y}(0) = (1, 0)$.

3. Consider the system $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$, where $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix}$.

(a) Find the eigenvalues of \mathbf{A} .

(b) Determine from the eigenvalues alone what type of equilibrium the system has at the origin.

(c) Find the general solution of the system.

(d) Find the solution of the system with the initial condition $\mathbf{Y}(0) = (1, 0)$.

- (e) Sketch the phase portrait, including the solution curve with the initial condition $\mathbf{Y}(0) = (1, 0)$.