Name: ______________________________

Math 224 Exam 4

March 30, 2015

1. Consider the system $\dfrac{d\mathbf{Y}}{dt} = \mathbf{AY}$, where $\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$.

   (a) Find the eigenvalues of $\mathbf{A}$.

   (b) Find the eigenvectors of $\mathbf{A}$.

   (c) Find the general solution of the system.

(d) Find the solution of the system with the initial condition $\mathbf{Y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(e) Sketch the phase portrait, including the solution curve with the initial condition $\mathbf{Y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

2. Consider the one-parameter family of linear systems given by

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} a & a+1 \\ a-1 & a \end{pmatrix} \mathbf{Y}.$$

(a) Sketch the corresponding curve in the trace-determinant plane.

(b) Identify which types of behaviors the system exhibits for which values of a.

3. Suppose a block with mass 1 is attached to the end of a spring with spring constant 5. The block is subject to a damping force proportional to its velocity, with a damping coefficient 4. Finally, an external time-dependent force of $\cos 2t$ acts on the block.

   (a) Write a differential equation which models the behavior of the block.

   (b) Find the general solution of your differential equation.

(c) Describe the long-term behavior of the block.

(d) Suppose that at time 0 the block is at rest and the spring is stretched so that the block is a distance 1 from its equilibrium position. Determine the position of the block for all times t.