Name: _____

Math 224 Exam 5
November 14, 2012

1. Consider the undamped, forced harmonic oscillator modeled by

$$\frac{d^2y}{dt^2} + 4y = 2\cos(\omega t).$$

(a) If $\omega \neq 2$, find the general solution to the equation above.

(b) If $\omega = 3$, give the solution to the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 2\cos(\omega t), \qquad y(0) = y'(0) = 0.$$

(c) Now suppose $\omega = 2$. Find the solution to the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 2\cos(\omega t), \qquad y(0) = y'(0) = 0.$$

(d) Describe the long-term behavior of your solution above, and give a rough sketch.

2. Suppose that countries X and Y are engaged in an arms race, and let $x(t)$ and $y(t)$ denote the respective sizes of the countries' stockpiles. Suppose the situation is modeled by the system of differential equations

$$\begin{aligned} \frac{dx}{dt} &= h(x,y) \\ \frac{dy}{dt} &= k(x,y). \end{aligned}$$

Suppose we have observed the following features in the rate of arms manufacture:

- If one county's stockpile in not changing, and the other increases its stockpile, it also slows its production rate.
- If either country increases its stockpile, the other responds by increasing its production rate.

(a) What do the assumptions imply about $\frac{\partial h}{\partial x}$, $\frac{\partial h}{\partial y}$, $\frac{\partial k}{\partial x}$, and $\frac{\partial k}{\partial y}$?

(b) What types of equilibria are possible for this system?

(c) Suppose that both countries don't slow their production much when they add to their stockpiles, but that their reactions to enemy stockpile growth are extreme (that is, they increase production rapidly in response to enemy stockpile growth). If there are any equilibria in the system, what sort would you expect?

(d) (*Extra Credit*) Can you tell what is likely to happen in the arms race?

3. Consider the non-linear system

$$\begin{aligned} \frac{dx}{dt} &= x(x-1) \\ \frac{dy}{dt} &= y(x^2-y). \end{aligned}$$

(a) Identify and sketch the nullclines (indicating the direction of the slope field along them).

(b) Classify the non-trivial equilibrium (i.e., the one for which neither x nor y is 0).