Name: ______________________________

# Math 224 Exam 5

April 17, 2015

1. (a) Solve the initial value problem $\dfrac{d^2y}{dt^2} + 4y = 3\cos 2t,\ y(0) = y'(0) = 0.$

   (b) Describe the long-term behavior of the solution.

2. Consider the system

$$\frac{dx}{dt} = y,$$
$$\frac{dy}{dt} = -y - \sin x.$$

(a) Find the nullclines and equilibria.

(b) Determine the types of the equilibria.
*Hint:* There are two sets of equilibria of different types.

(c) Sketch the phase portrait for the system, including at least three equilibria.

3. Consider the system

$$\frac{dx}{dt} = 2xy,$$
$$\frac{dy}{dt} = -y^2.$$

(a) Show that the system is Hamiltonian.

(b) Find a Hamiltonian function $H(x, y)$ for the system.

4. Solve the initial value problem $\dfrac{dy}{dt} + 7y = u_2(t)$, $y(0) = 3$.

# Formulae

$$\mathcal{L}[y] = \int_0^\infty y(t)e^{-st}\ dt$$

$$\mathcal{L}[y'] = s\mathcal{L}[y] - y(0)$$

$$\mathcal{L}[y''] = s^2\mathcal{L}[y] - sy(0) - y'(0)$$

| $y(t)$ | $Y(s) = \mathcal{L}[y]$ |
|---|---|
| $1$ | $\frac{1}{s}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| $u_a(t)$ | $\frac{e^{-sa}}{s}$ |
| $u_a(t)f(t-a)$ | $e^{-as}F(s)$ |
| $e^{at}f(t)$ | $F(s-a)$ |
| $tf(t)$ | $-\frac{dY}{ds}$ |