Name: ______________________________________ Group Number:__________

Math 224 Exam 1

September 16, 2019

1. (a) Solve the initial value problem

$$\frac{dy}{dt} = \frac{1}{1-y}, \qquad y(0) = 0.$$

(b) What is the domain of definition of your solution in part (a)? Describe as precisely as possible what happens as t approaches the upper limit of the domain, and sketch your solution.

2. Suppose a population of rabbits satisfies the logistic growth model:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right),$$

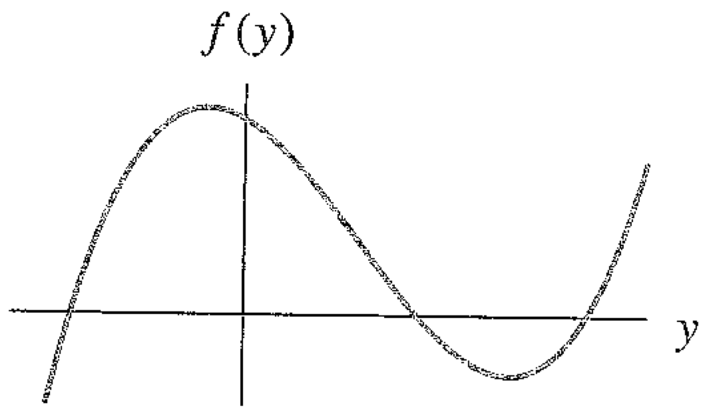
where k is the growth constant and N is the carrying capacity.

(a) Sketch the phase line and classify the equilibria of the differential equation. Use the phase line to sketch representative solutions of the differential equation (all on the same axes).

(b) Now suppose that additional rabbits migrate into the area at a steady rate of α rabbits per unit time. Modify the equation to reflect this.

(c) How do the equilibria of the system change based on the migration?

3. Suppose the following is a graph of the function $f(y)$.



Sketch the slope field of the differential equation

$$\frac{dy}{dt} = f(y),$$

and describe the possible long term behaviors of the solutions, depending on their initial conditions.

(**Suggstion**: label important points on the axes of the graph above.)

4. Consider the differential equation

$$\frac{dy}{dt} = -y + 2e^{-t}.$$

(a) Show that $y_1(t) = (2t - 1)e^{-t}$ and $y_2(t) = (2t + 1)e^{-t}$ are both solutions

(b) What does the uniqueness theorem say about a solution with initial condition $y(0) = 0$?

(c) Find the solution to the initial value problem

$$\frac{dy}{dt} = -y + 2e^{-t} \qquad y(0) = 0$$

and confirm that your claim in part (b) was true.