Name: ________________________________

Math 224 Quiz 2 – E. Meckes

1. Consider the model for the damped harmonic oscillator given by

$$y''(t) + 2y'(t) + 10y = 0.$$

   (a) Show that $y_1(t) = e^{-t}\sin(3t)$ and $y_2(t) = e^{-t}\cos(3t)$ are both solutions to the differential equation above.

   (b) Convert the second-order equation into a first-order system. What solutions to the system correspond to the solutions you were given to the second-order equation?

(c) Give the general solution (to either the system or the second-order equation).

(d) Describe the typical long-term motion of the block in this model.

2. Recall the basic SIR Model of an epidemic:

$$\frac{dS}{dt} = -\alpha IS \qquad \frac{dI}{dt} = \alpha SI - \beta I \qquad \frac{dR}{dt} = \beta I,$$

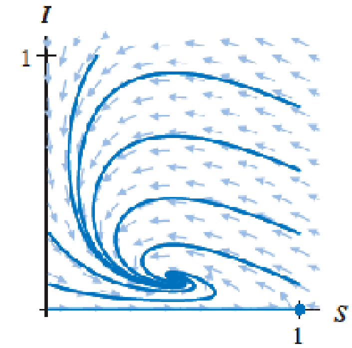
where S is the portion of the population that is susceptible, I is the portion infected, and R is the portion "recovered"; i.e., not infected or susceptible.

Suppose now that the disease is evolving so that recovered people become susceptible to new strains at a rate proportional to the size of the recovered population.

(a) Modify the basic model to reflect this.

(b) Give a two-dimensional version of the new model, involving only S and I. (Recall that $S(t) + I(t) + R(t) = 1$ for all t.)

(c) Here is a picture of the phase plane of the two-dimensional model (for a particular choice of parameters). If the disease is initially introduced into the population by a small number of people, what happens in the long-term?



3. Consider the linear system $\frac{d\mathbf{Y}}{dt} = \mathbf{B}\mathbf{Y} = \begin{bmatrix} -2 & -3 \\ -3 & -2 \end{bmatrix} \mathbf{Y}$.

(a) Find the eigenvalues of B.

(b) Find the corresponding eigenvectors.

(c) Give the general solution to the system.

(d) Solve the initial value problem $\frac{d\mathbf{Y}}{dt} = \begin{bmatrix} -2 & -3 \\ -3 & -2 \end{bmatrix} \mathbf{Y}$ and $\mathbf{Y}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

(e) What is the long-term behavior of your solution as $t \to \infty$? What about $t \to -\infty$?

(f) Sketch the phase plane for this system. Make sure to include any straight-line solutions, indicate direction of solution curves in time, and include the solution curve you found above to the initial value problem.