Name: _____

Math 224 Quiz 3 – E. Meckes

1. Consider the non-linear system

$$\frac{dx}{dt} = y - ax^3$$
$$\frac{dy}{dt} = y - x,$$

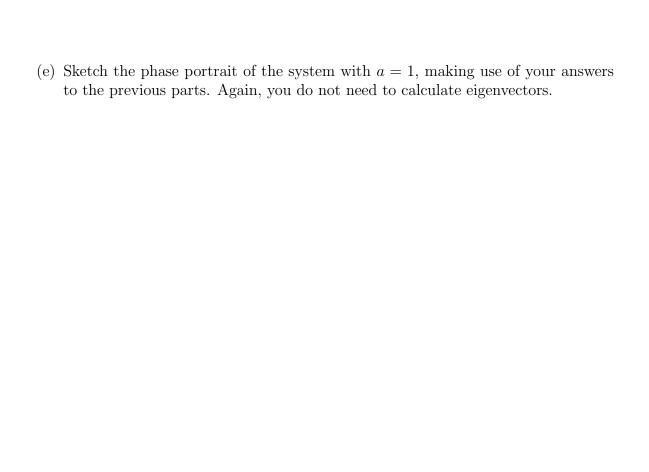
where a is a parameter.

(a) Identify the nullclines of the system. Then find all the equilibria of the system, and identify the value(s) of a at which the number and/or location of the equilibria changes.

(b) Sketch the nullclines for a = -1, a = 0, and a = 1. You do not need to draw arrows, just the nullclines themselves. Please label everything in your pictures!

(c) For a=1, compute the Jacobian of the system.

(d) Linearize the system at each of the equilibria and identify the type of equilibrium. You do not need to calculate eigenvectors.



2. Consider the undamped, forced harmonic oscillator modeled by

$$\frac{d^2y}{dt^2} + 4y = 2\cos(\omega t).$$

(a) If $\omega \neq 2$, find the general solution to the equation above.

(b) If $\omega = 3$, give the solution to the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 2\cos(\omega t), \qquad y(0) = y'(0) = 0.$$

(c) Now suppose $\omega=2$. Find the solution to the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 2\cos(\omega t), \qquad y(0) = y'(0) = 0.$$

(d) Describe the long-term behavior of your solution above, and give a rough sketch.